C

SIGMAONE

June 5, 2001

EXHIBIT C

"MERITS AND LIMITATIONS OF RSSI/MAHO LOCATION DETERMINATION"

THEORETICAL ANALYSIS AND FIELD TEST RESULTS

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1. OVERVIEW

1.1. Background

Over the past several years SigmaOne Communication has thoroughly investigated various network-overlay location technologies and techniques. These techniques included Angle Of Arrival (AOA), Time Difference Of Arrival (TDOA), Time Advance (TA) and techniques using Received Signal Strength (RSSI). For the purpose of these investigations, SigmaOne conducted extensive theoretical studies, laboratory and field tests in order to evaluate and select the ultimate technology it would use to address the FCC E-911 requirement.

The experience gathered in all these tests shows that for narrowband systems, and specifically for AMPS and TDMA, AOA is the most accurate technology and TDOA rates second. Received Signal Strength Indication (RSSI) was judged to be too erratic in its behavior to be relied upon to provide any reasonable location accuracy. Subsequent to these findings, SigmaOne concluded that for AMPS and TDMA, a combination of the first two technologies will be required to meet both the FCC's stringent E911 location requirements and the demands of wireless carriers for future commercial location based services.

1.2. Overview of RSSI/MAHO based location techniques

Received signal strength is the difference between transmitted power and propagation path loss. One may assume that in a well documented network deployment transmitted power is typically known with reasonable accuracy. On the other hand, propagation path loss fluctuates both in time and in space within a wide range of values. Path loss is measured on a logarithmic scale in dB. Path loss is typically a product of signal attenuation due to range and other unpredictable factors as shown below. RSSI techniques rely on the location system's capability to map RSSI measurements to location.. As we will see below, path loss is in most cases quite unpredictable and could therefore sway the perceived distance to the handset by a significant factor. For example, in a typical Suburban/Urban setting, nominal (albeit unpredictable) fluctuations of 8-10 db in the path loss may be interpreted by the location system as a up to a 100% variation in the range to the handset.

There are at least four factors, extremely difficult to predict, that prevent signal strength, especially handset based measurements of RSSI, from being usable for the purpose of position determination:

1. **Shadowing** - Widely documented in technical literature, wireless propagation path loss varies between 20 dB per decade for line of sight transmissions, to 50 dB in directions when there is heavy shadowing. Even outdoors shadowing varies significantly when moving around the corner of a building, between a cross street and when crossing a street. Variations in signal strength inside a vehicle are typically 10 dB when moving from one window to another; obviously reception of another site may vary in the opposite direction under the same circumstances, leading to huge perceived shifts in the estimated handset location. Variations between received signal strength from different sites when inside a building are even greater. Moreover, within buildings reception may be limited to one or two sites, rendering it impossible to determine position by means of this technique.
2. **Multipath**. Changes of location in the order of the signal wavelength, 12 inches or less may cause variations of signal strength of up to 30 dB due to the effect of multipath. Obviously, reception from different sites is affected in different ways, leading to even greater errors in the estimated location .
3. **Handset RSSI**. Handsets are required to measure received signal strength **in** a monotonous function to be able to decide which site is a better candidate for service. The requirement for absolute accuracy, specified by TIA/EIA-136-123-B, allow for RSSI measurement errors of up to 10 dB. Also, the reported measurements are very coarse using 2 dB steps. Although vendors have taken different approaches to RSSI implementation leading to better figures than required by the standards, the use of the handset as a "coarse" signal strength measurement tool significantly increases the error contributed by varying propagation conditions. Moreover, the handset is not capable of reporting receive conditions when interference due to frequency reuse degrades the quality of signal reception - an effect often encountered in field tests that is only overcome with sophisticated signal processing only available in special position determination equipment.
4. **Handset antenna directionality**. Unless held in a vertical position far from interfering objects, the antenna gain in different directions, hence towards different cell sites, varies significantly. The variation is impacted by the angle the phone is held as well as coupling: how the handset is held and the relative location of nearby objects and the materials they are made of such as metal and windows inside cars, building walls and windows, nearby parked cars and trucks, etc.

Due to the factors described above, even in cases when handset location varies within a relatively small area, many parameters will severely impact the measured RSSI, materially affecting the perceived distance to the handset.

1.3. Summary Highlights

This exhibit includes a theoretical analysis of RSSI/MAHO based location, numerical computer predictions of such a location system and results from a number of field tests recording handset RSSI measurements and using those measurements as the basis for the evaluation of accuracy performance.

1.3.1. Theoretical Limits:

SigmaOne has described many of the significant issues associated with making accurate signal strength measurements in a real world environment. These issues include antenna orientation, multipath, co-channel interference and path loss. By utilizing a simulation tool implementing the accuracy estimation models developed in paragraph 5 below, and by assuming typical and widely accepted RSSI standard deviation values of 11.8 dB, for a 12 cell site area in which the maximum separation between sites is less than 1 mile, we show that the typically achievable location error is in the order of 1000-1500 meters/67%. As the separation grows to more realistic values, we expect the errors to grow as well.

1.3.2. Test Results

SigmaOne summarized data recorded from a total of 12 tests that were performed in order to verify the theoretical performance of using RSSI/MAHO measurements as the basis for location. The overall error statistics achieved in those tests are summarized in Table 1. Our tests show a 922 meter/67% error and a 2112 meter/95% error overall. The individual results of the 12 tests are summarized in Table 2.

Table 1 - Overall Test Summary

Location Error Summary	All Tests
67% of range errors were below:	922 meter
95% of range errors were below:	2112 meter

Table 2 - Individual Test Summaries

	Test No. 1	Test No. 2	Test No. 3	Test No. 4	Test No. 5	Test No. 6
67% of range errors were below:	**760 meter**	**810 meter**	**1008 meter**	**1264 meter**	**680 meter**	**745 meter**
95% of range errors were below:	**2100 meter**	**3276 meter**	**2627 meter**	**2203 meter**	**1455 meter**	**1442 meter**

	Test No. 7	Test No. 8	Test No. 9	Test No. 10	Test No. 11	Test No. 12
67% of range errors were below:	**854 meter**	**951 meter**	**803 meter**	**888 meter**	**1027 meter**	**1277 meter**
95% of range errors were below:	**2538 meter**	**2696 meter**	**1458 meter**	**2040 meter**	**2046 meter**	**3442 meter**

1.3.3. Conclusion

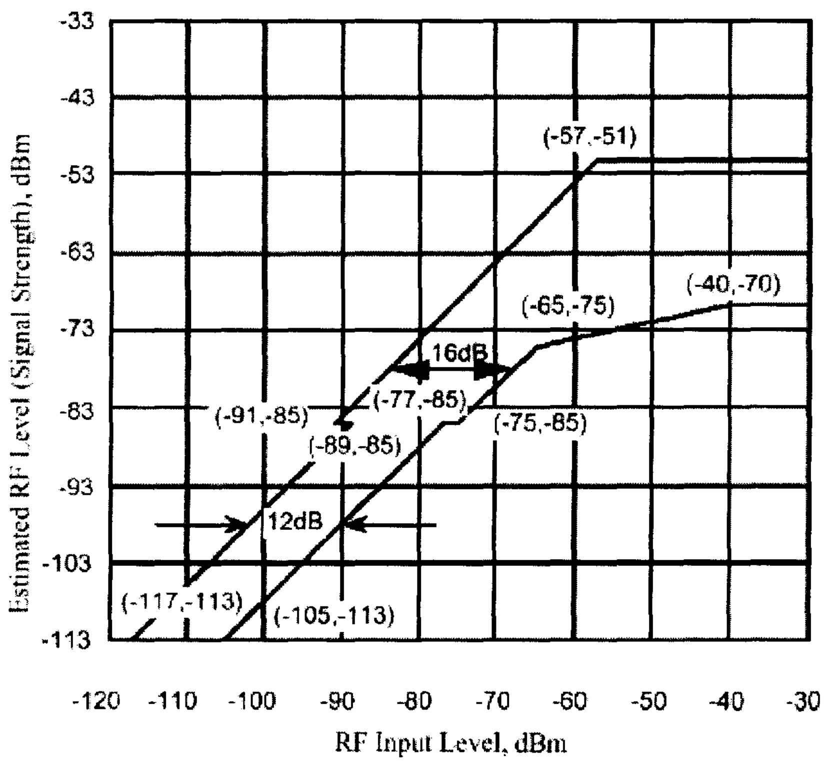
It is quite clear from our test results and the performance analysis conducted on RSSI/MAHO based location determination, that this technique will never satisfy the commission's rules for accuracy and will perform well below the AT&T stated accuracies of 250m-67% and 750m-95%.

2. RSSI/MAHO BASED LOCATION

2.1. RSSI in IS-136

According to TIA/EIA-136-123-B (March 2000) and previous versions of IS-136, a Control Channel Selection procedure is executed in order to allow a mobile station to determine whether or not a given candidate control channel is acceptable for camping purposes. Upon finding a candidate control channel not marked as ineligible, a mobile station shall proceed to execute the Signal Strength Aspects Determination procedure and the Service Aspects Determination procedure.

"Signal strength measurements shall be made with an absolute accuracy determined according to the following figure"1.



1 TIA/EIA-136-123-B-2000, March 31, 2000, pages 66-67

The relative accuracy, defined as the error in dB, between the difference of the two estimated RF levels and the difference between the corresponding RF input levels, is ± 3 dB in the range -105 to -85 dBm of estimated RF levels. In addition, if the estimated RF levels spans a wider range, from -105 to -75 dBm, the relative accuracy shall be ± 5 dB.

The received signal strength measurements are used by the MNLS method for determining the mobile station location. Note that if, for example, the estimated (reported) signal strength is -113dBm the actual signal strength is between -117dBm and -105 dBm (the error is bounded to the interval [-4dB, +8dB]). For higher signal levels the accuracy requirement is relaxed. If for example, the reported signal strength is -85dBm the actual signal strength is between -91dBm and -75dBm (the error is bounded to the interval [-6dB, +10dB]). These are rather large absolute errors and they reflect the difference in reporting between different handsets that are collocated and receive the same signals. Even if the handset bias is somehow removed its measurement error is limited to only 3-5dB.

2.2. Signal Strength Issues

A multitude of parameters affect the received signal strength at the handset. These parameters include:

1. Cell-Site Transmitted Power
2. Cell-Site Transmit Antenna three dimensional radiation pattern
3. Cell-Site Transmit Antenna orientation (azimuth and tilt)
4. Cell-Site Transmit Antenna height above ground level
5. Handset Antenna three dimensional radiation pattern
6. Handset Antenna
7. Orientation (polarization)
8. Handset Antenna height above ground level
9. Multi-path and Fading
10. Co-channel Interference
11. Path-loss

Note: Although the first 4 factors could be measured, documented and stored in carriers OSS database, creating and maintaining such a data base for all of carrier's cell sites seems to be quite impractical. However, no location system can assume the following parameters to be known:

1. Receive antenna three dimensional radiation pattern (Is not identical for different handset models.)
2. Receive antenna orientation (Different people hold the phone in different position: near ear, away from body using earphones, within a car, etc.)
3. Receive antenna height above ground level (Different floors of same building.)
4. Multi-path and fading (Depends on phone location and RF reflectors and scatterers.)
5. Co-channel interference (Depends on man-made noise, frequency reuse, etc.)
6. Path-loss affected by season, foliage, climate, etc.

3. RSSI/MAHO BASED LOCATION TECHNIQUES – PERFORMANCE ESTIMATION BASED ON FIELD TRIALS

SigmaOne has conducted a number of tests in order to assess the reliability of using the handset reported RSSI measurements for estimating the handset location. This section presents the measurements, a summary analysis and conclusions.

3.1. Description of First Test

A standard TDMA cellular phone (Ericsson TEMS KH668) was connected to a laptop computer in order to record the phone reported RSSI measurements.

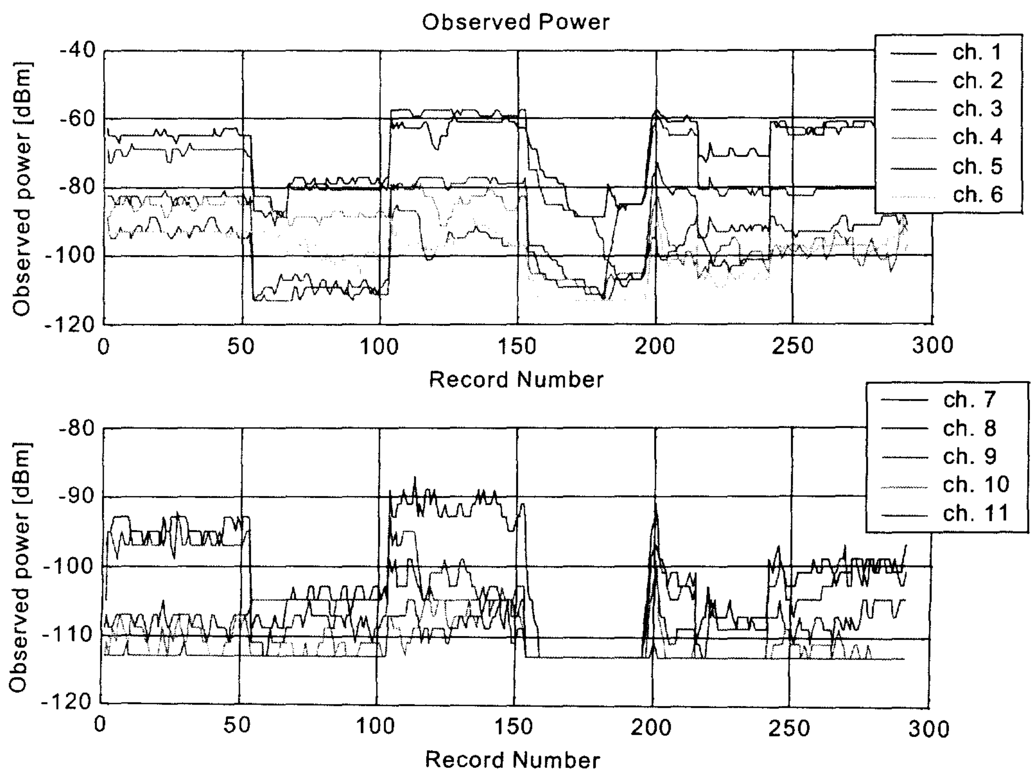
The phone was tested in 5 different scenarios:

1. Phone placed on a table in room No. 1
2. Phone placed on a table in room No. 2 (adjacent to room 1)
3. Room No. 1, phone adjacent to left side of human head
4. As in 3 above, but antenna touches head
5. Room No. 1, phone adjacent to right side of human head

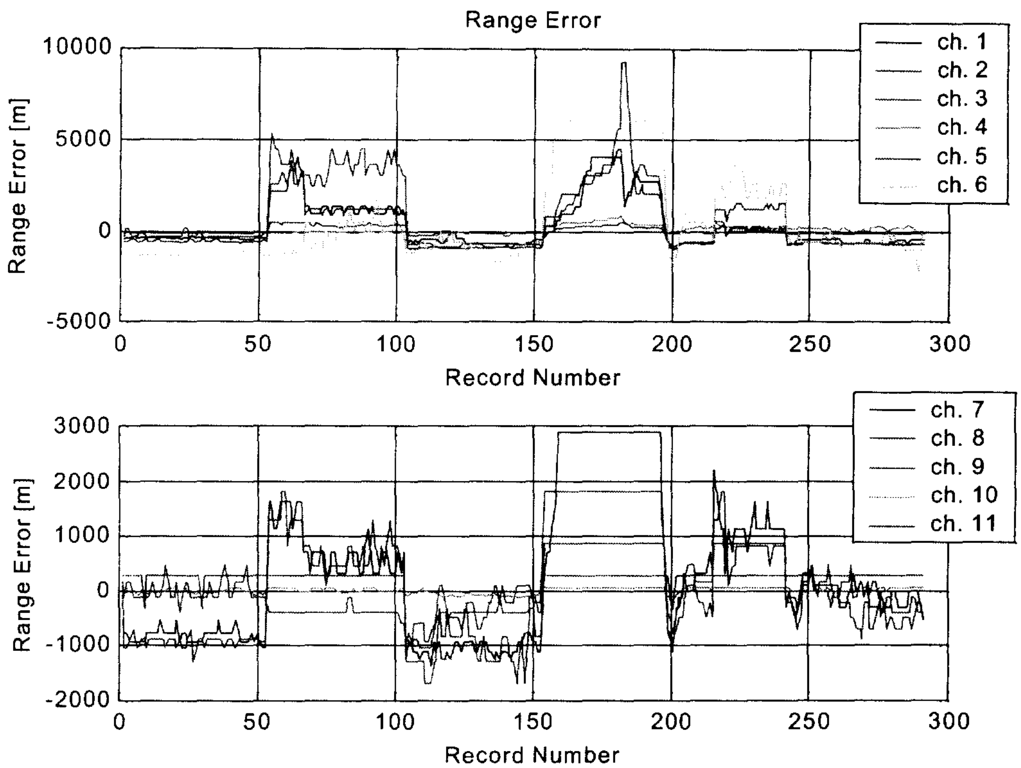
The handset intercepted, most of the time, 11 distinct control channels, associated with 6 distinct base stations each having three sectors.

Channel Serial Number	Control Channel No.	Base Station No.	Distance from handset [meter]
1	646	1	1400
2	750	1	1400
3	762	**1**	**1400**
4	661	**10**	**300**
5	781	**10**	**300**
6	798	**2**	**3800**
7	771	**16**	**2200**
8	752	**16**	**2200**
9	766	**15**	**2000**
10	796	**10**	**300**
11	746	**13**	**4300**

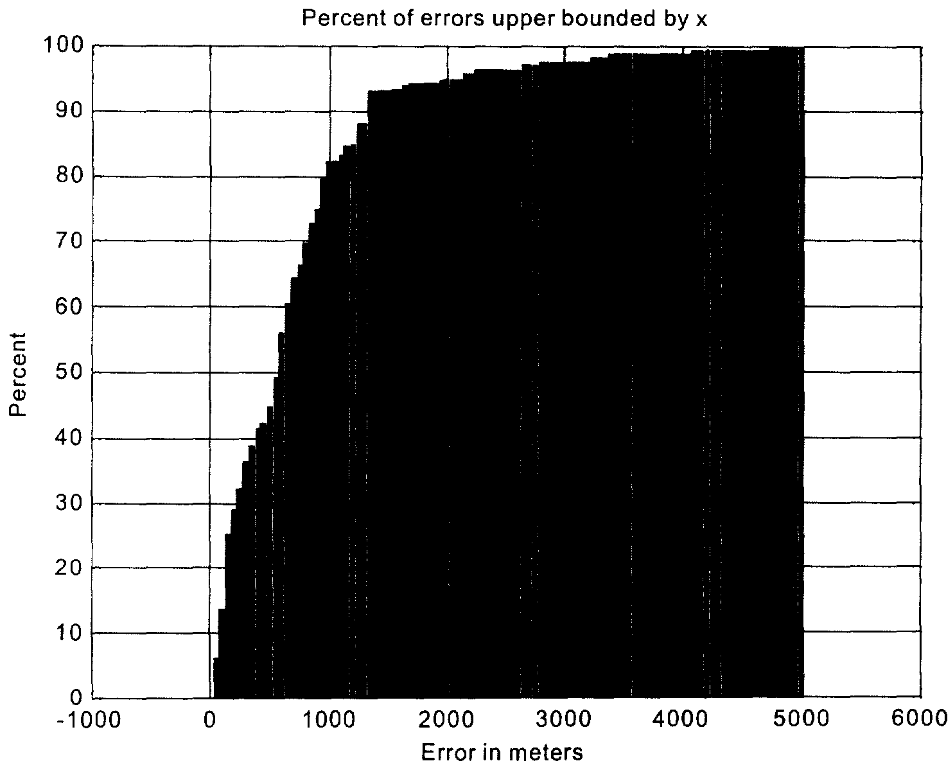
The following figure shows the reported signal strength as a function of time.



The above results correspond to range errors as shown in the following plot.

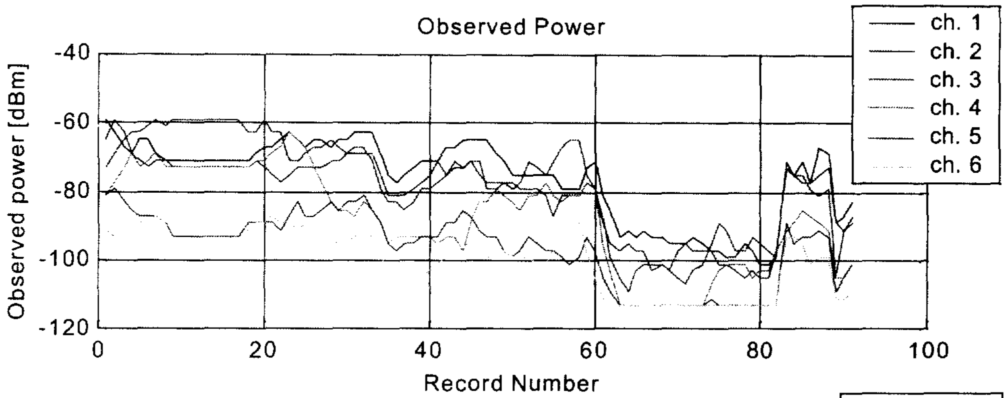


The following plot summarizes the error statistics.



3.2. Description of Second Test

A standard TDMA cellular phone (Ericsson TEMS KH668) was connected to a laptop inside a car, which moved slowly within a 50 x 50 meters parking lot.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Observed power [dBm]
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-60
-80
-100
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20
40
60
80
100
Record Number
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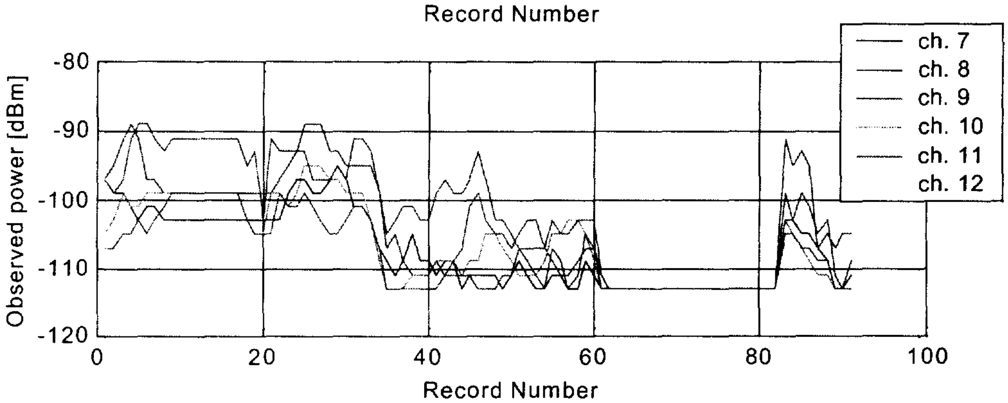
ch. 2
ch. 3
ch. 4
ch. 5
ch. 6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60
80
100
Record Number
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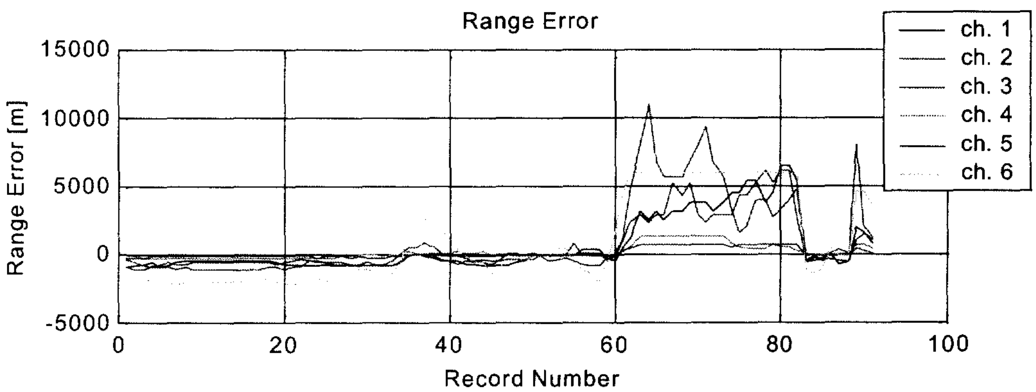
ch. 10
ch. 11
ch. 12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Range Error [m]
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10000
5000
0
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0
20
40
60
80
100
Record Number
ch. 1
ch. 2
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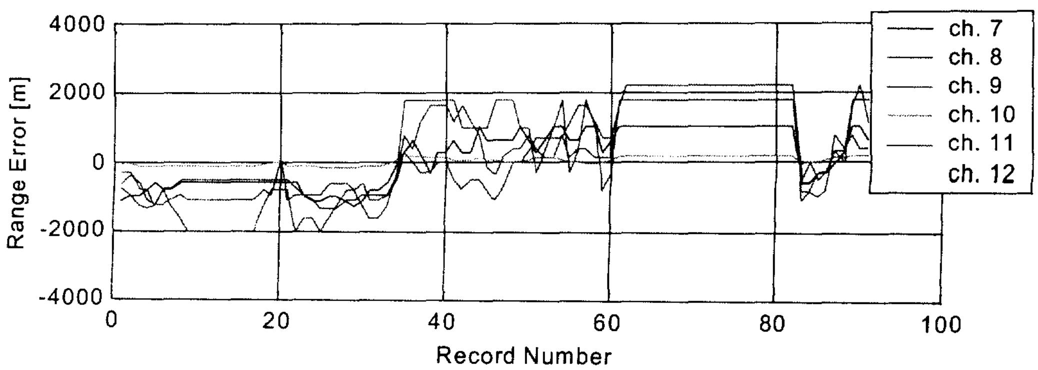
ch. 4
ch. 5
ch. 6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100
Record Number
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ch. 8
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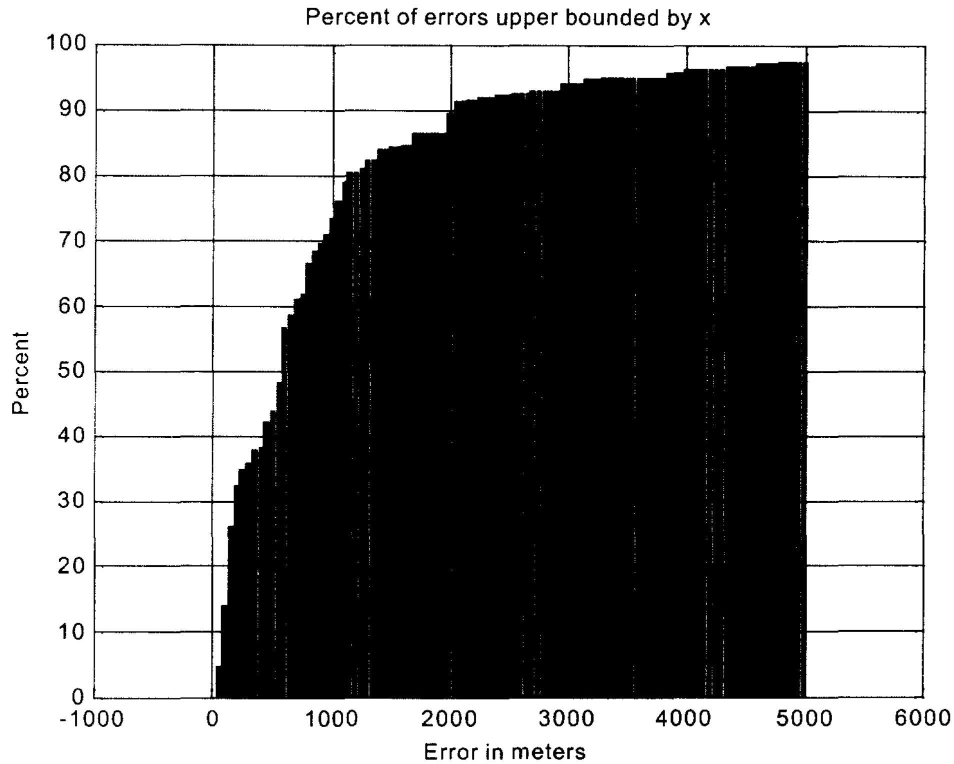
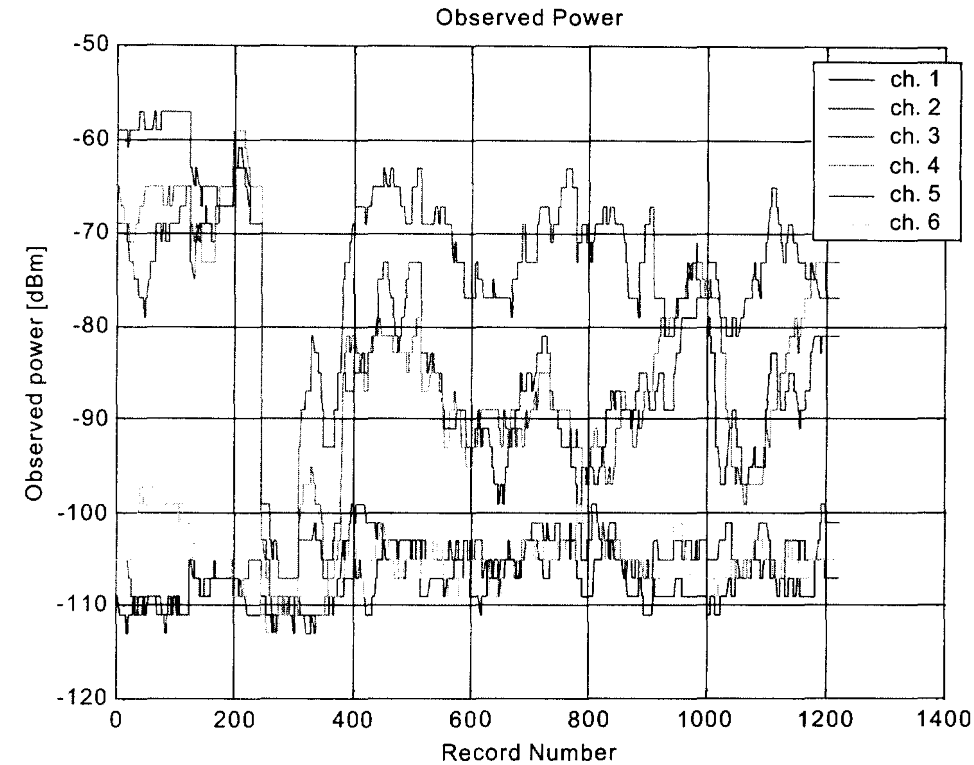
ch. 10
ch. 11
ch. 12



3.3. Description of Third Test

The third test was performed within an industrial park.


Observed Power
Observed power [dBm]
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-90
-100
-110
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1000
1200
1400
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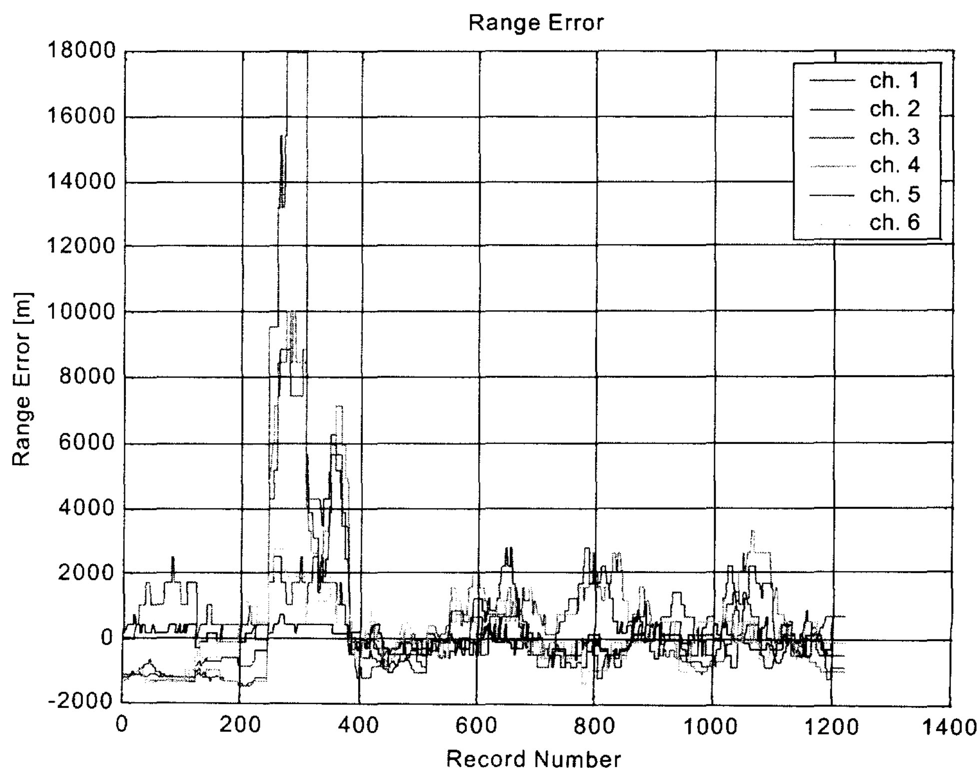
ch. 3
ch. 4
ch. 5
ch. 6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Range Error [m]
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16000
14000
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10000
8000
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4000
2000
0
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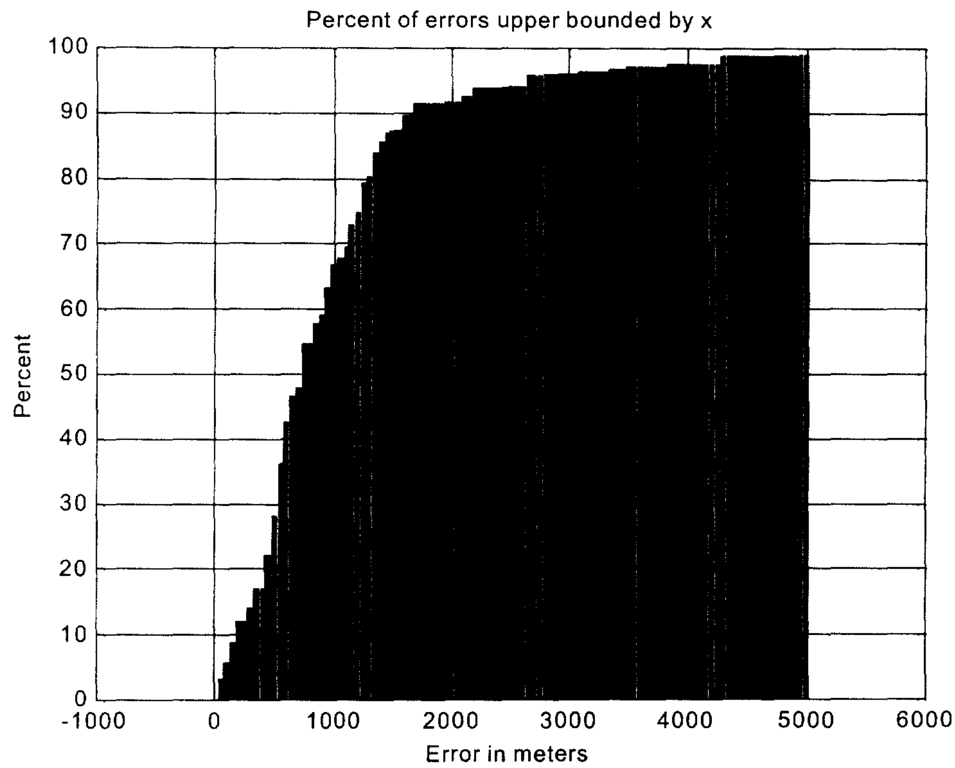
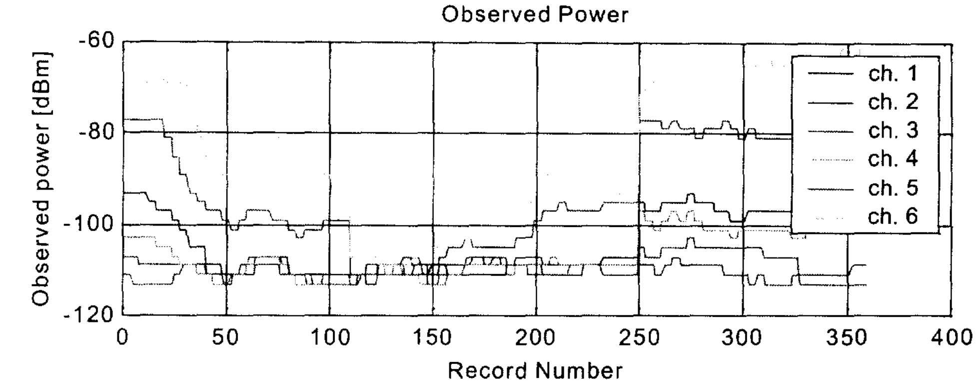
1200
1400
Record Number
ch. 1
ch. 2
ch. 3
ch. 4
ch. 5
ch. 6



3.4. Description of Fourth Test

The fourth test was performed within an industrial park.


Observed Power
Observed power [dBm]
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-80
-100
-120
0
50
100
150
200
250
300
350
400
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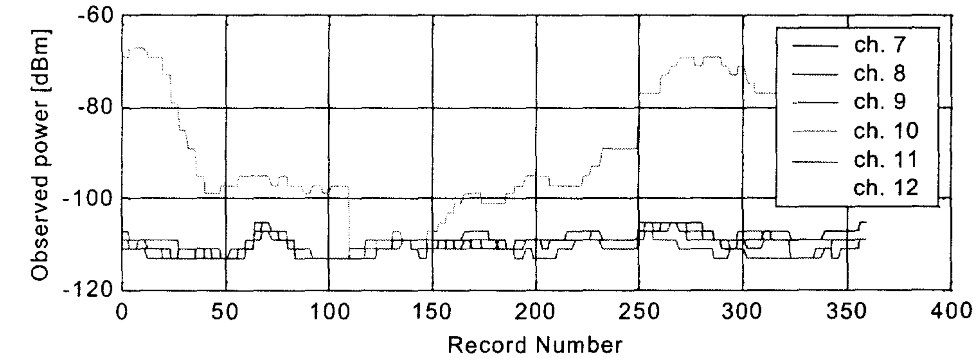
ch. 4
ch. 5
ch. 6


Observed power [dBm]
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-120
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250
300
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Record Number
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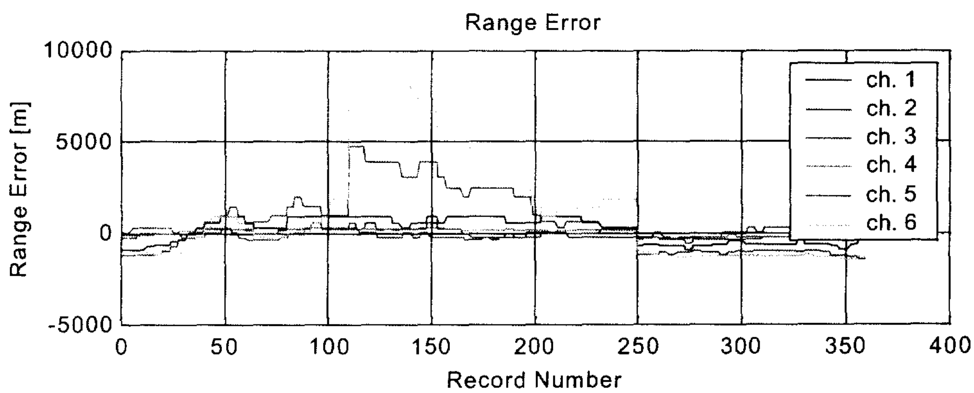
ch. 10
ch. 11
ch. 12


Range Error
Range Error [m]
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5000
0
-5000
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Record Number
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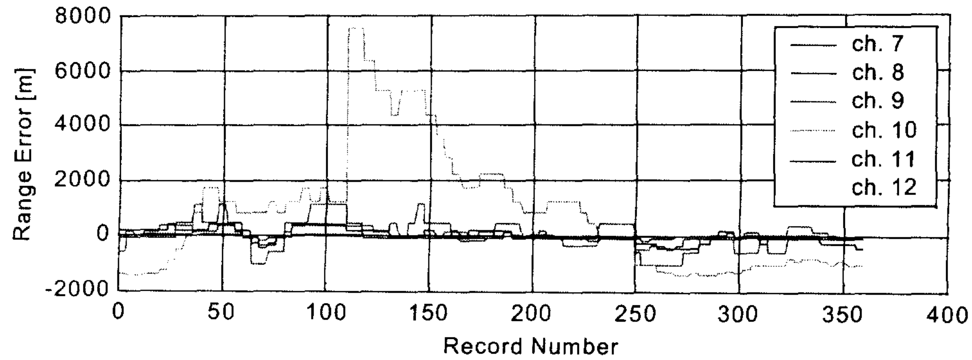
ch. 4
ch. 5
ch. 6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6000
4000
2000
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0
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400
Record Number
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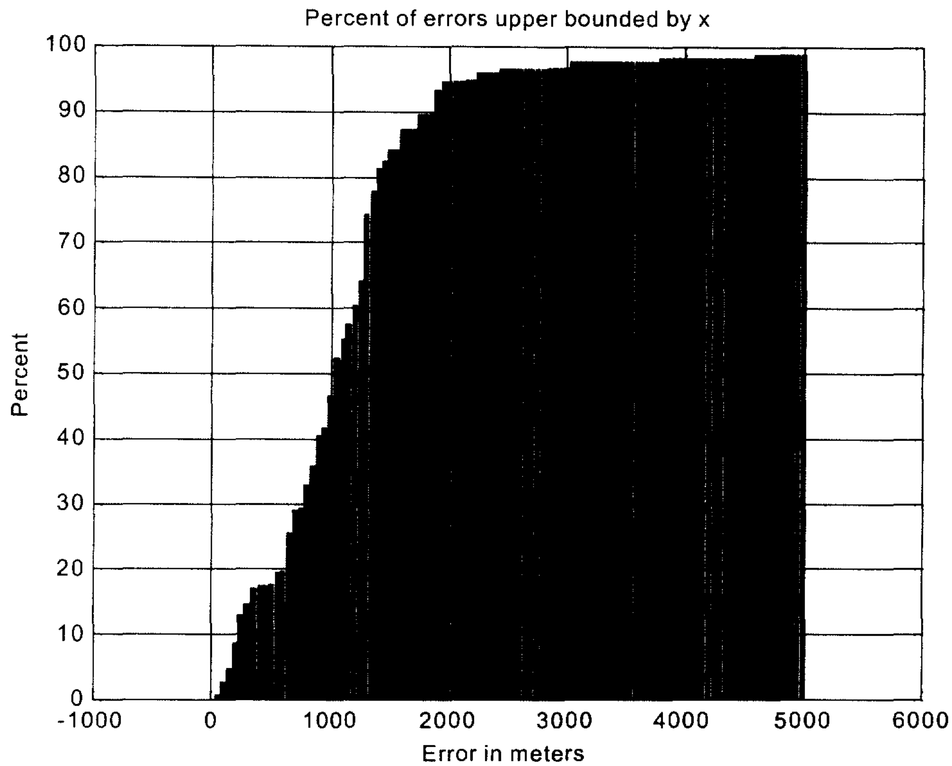
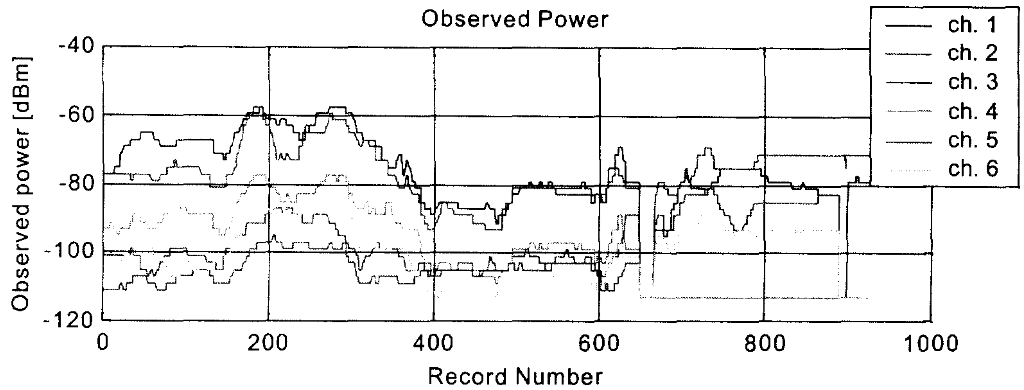
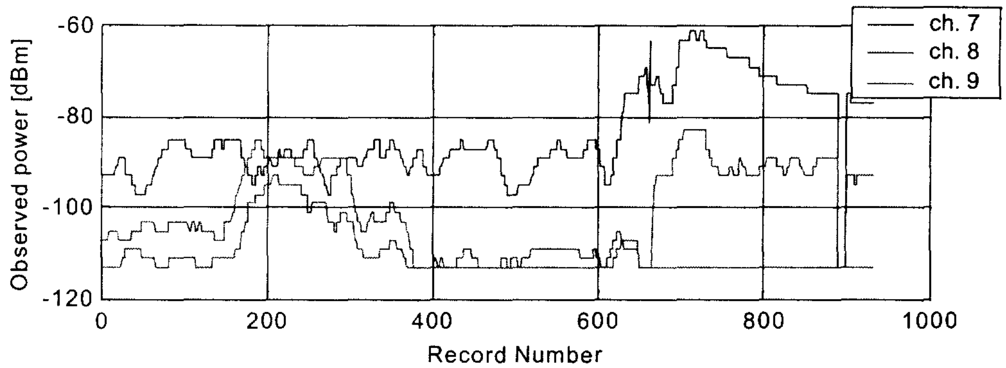
ch. 10
ch. 11
ch. 12



3.5. Description of Fifth Test

This test took place in SigmaOne's second floor offices on the second floor. The RSSI messages were recorded while the handset was located in three adjacent rooms: The first room is facing the east and has line of sight to a base station located 1400m from the building. Then, the handset was placed in a nearby room, located at the center of the second floor. In the 3rd step the handset was placed in a room facing the west with line of sight to a base station located 300m from the building. The distance between the three rooms is less than 20m.


Observed Power
ch. 1
ch. 2
ch. 3
ch. 4
ch. 5
ch. 6
Observed power [dBm]
-40
-60
-80
-100
-120
0
200
400
600
800
1000
Record Number


ch. 7
ch. 8
ch. 9
Observed power [dBm]
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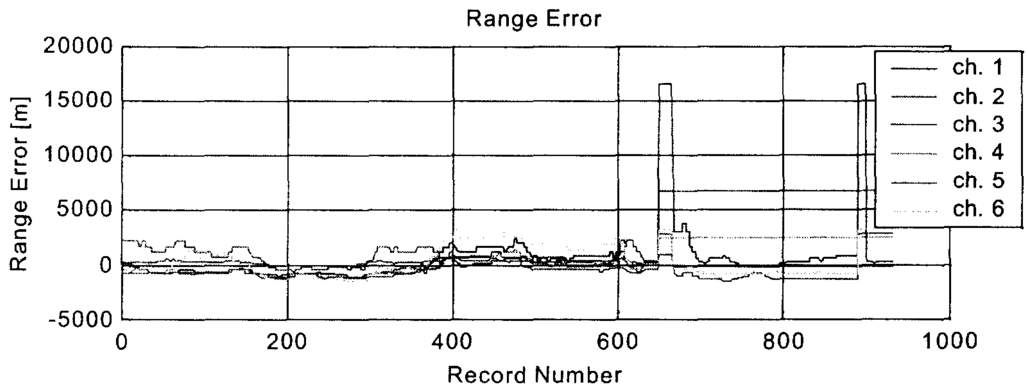
-80
-100
-120
0
200
400
600
800
1000
Record Number


Range Error
ch. 1
ch. 2
ch. 3
ch. 4
ch. 5
ch. 6
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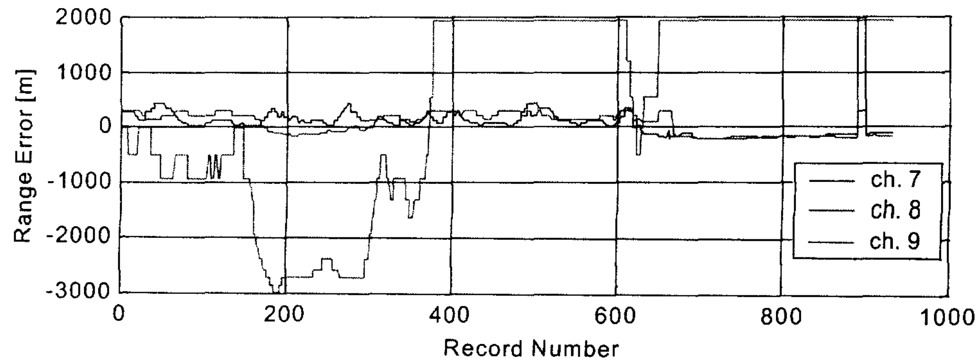
400
600
800
1000
Record Number


ch. 7
ch. 8
ch. 9
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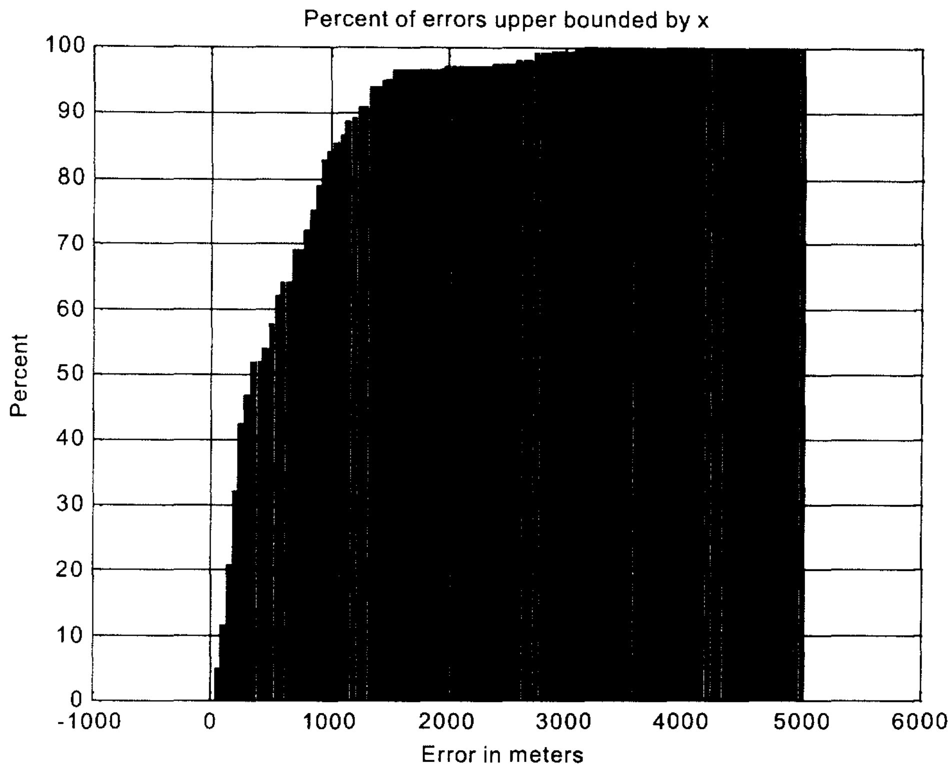
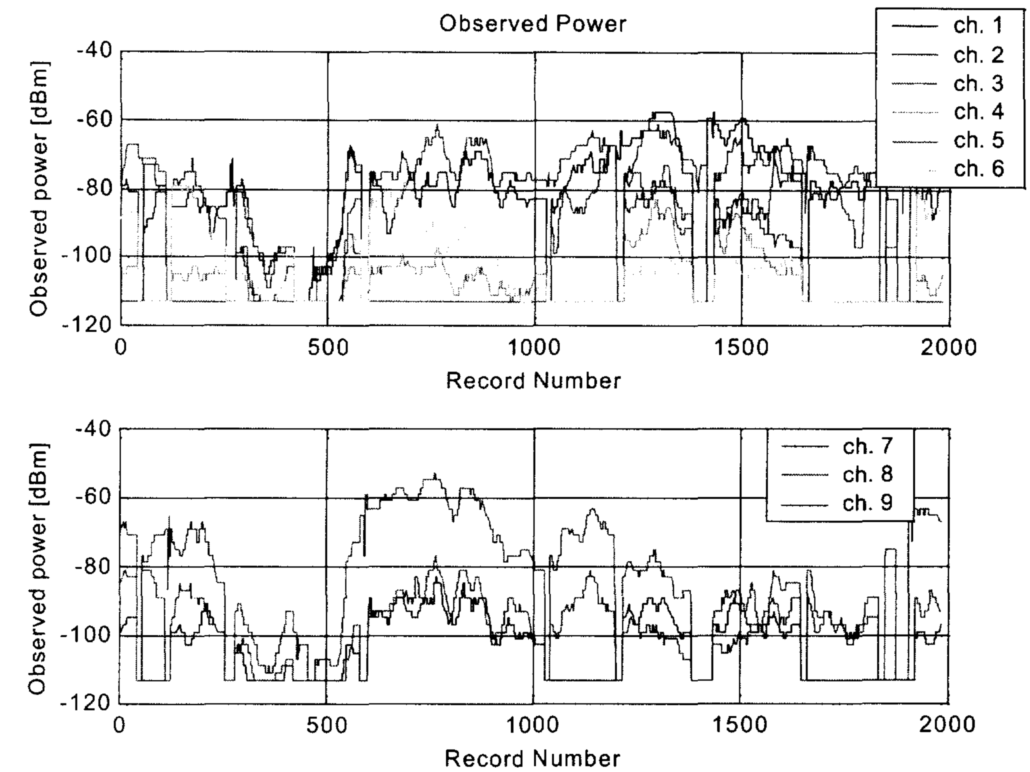
-3000
0
200
400
600
800
1000
Record Number

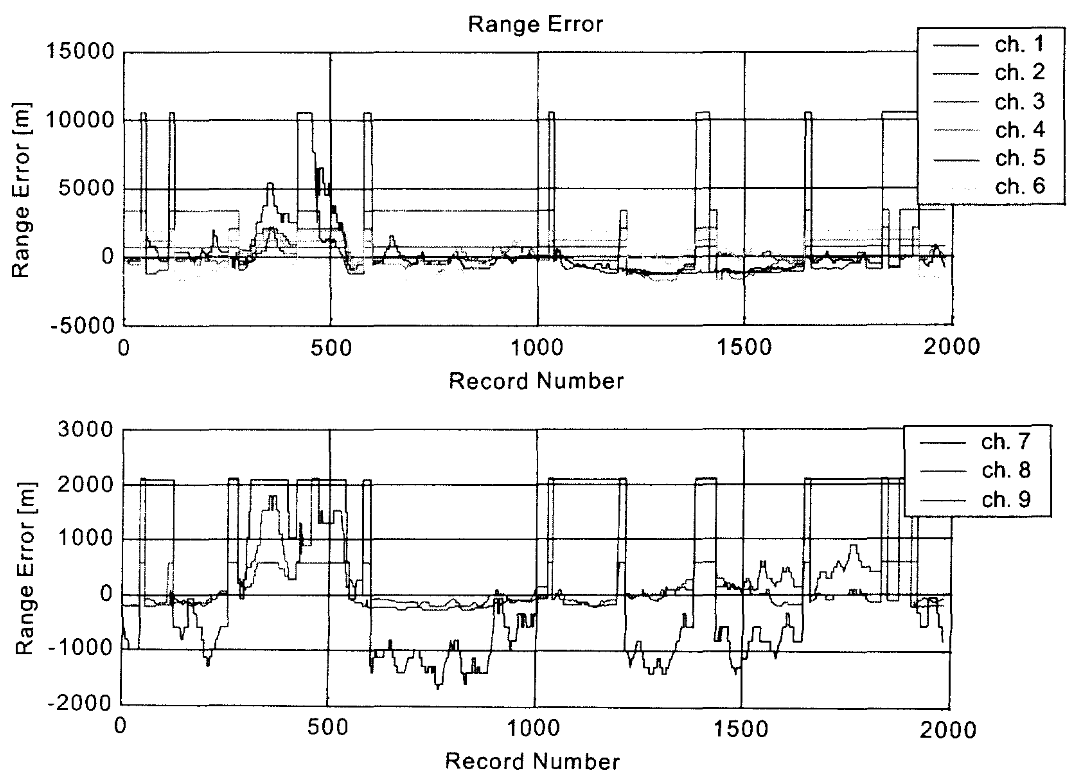


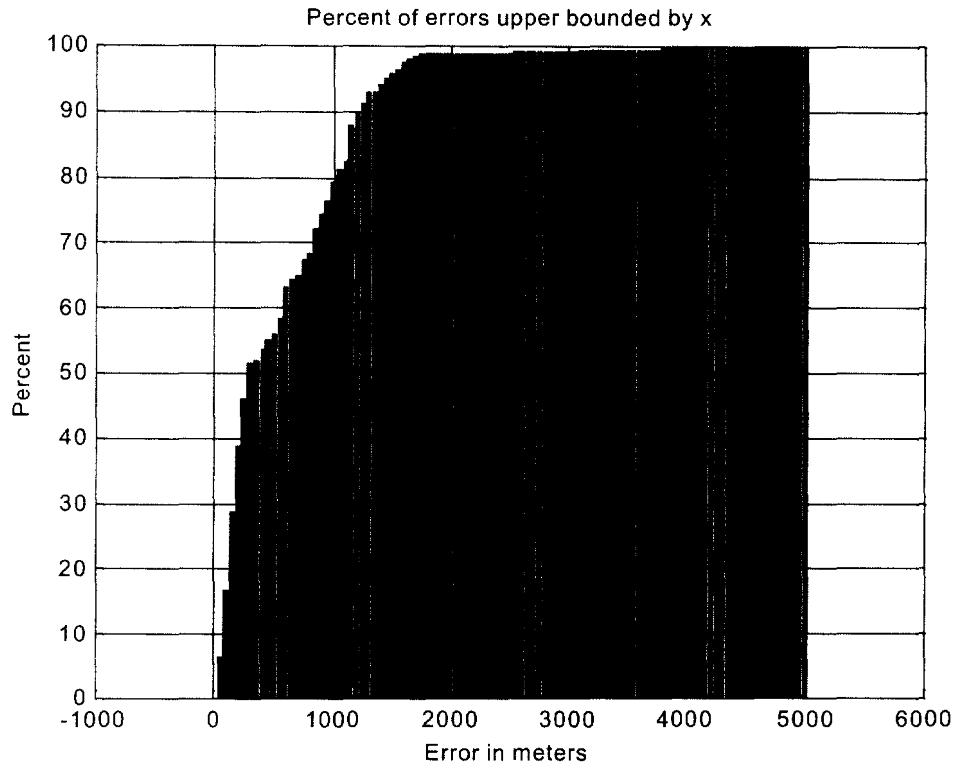
3.6. Description of Sixth Test

This test took place in SigmaOne's offices in Rehovot, Israel. The RSSI messages were recorded while the handset receiver was carried indoor and outdoor. We started in our second floor office, then using the elevator went up to the fifth floor, then down to the ground floor and around the building.

The building has 5 floors, its width is 25 m and its length is 75m.


Observed Power
Observed power [dBm]
Record Number
ch. 1
ch. 2
ch. 3
ch. 4
ch. 5
ch. 6
ch. 7
ch. 8
ch. 9


Range Error
Range Error [m]
Record Number
ch. 1
ch. 2
ch. 3
ch. 4
ch. 5
ch. 6
ch. 7
ch. 8
ch. 9

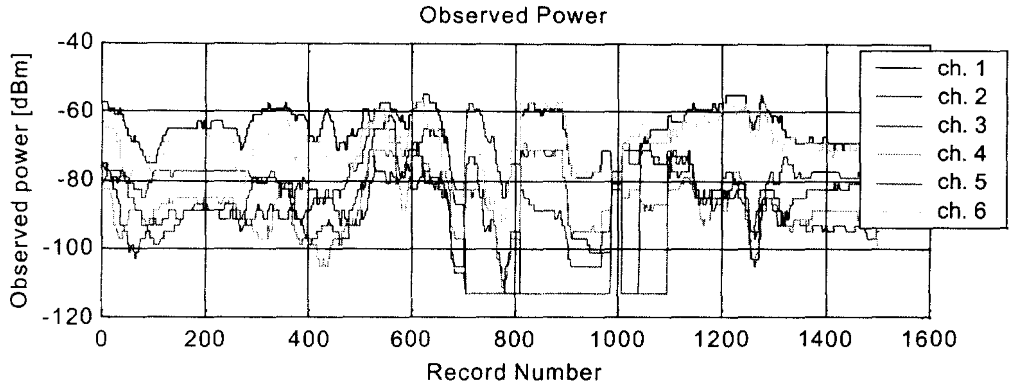


3.7. Description of tests 7 to 10

This test took place in the Nes-Ziona Mall, a shopping center in Nes-Ziona, Israel. All the Nes-Ziona Mall trials were recorded within an area of 30m radius. Four sets of data were recorded :

1. The first set was recorded at the second floor of the parking garage.
2. The second set was recorded at the second floor of the shops section.
3. The third set was recorded at the ground floor of the shops section.
4. The fourth set was recorded at the ground floor of the parking garage.

Test # 7


Observed Power
Observed power [dBm]
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-60
-80
-100
-120
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200
400
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1000
1200
1400
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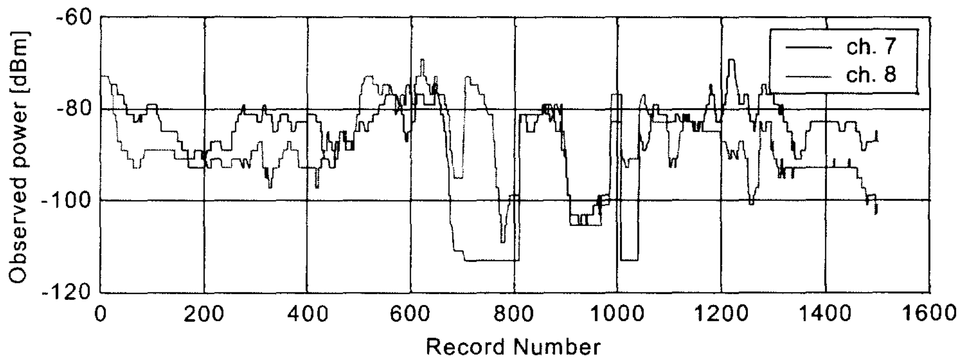
ch. 4
ch. 5
ch. 6


Observed power [dBm]
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-80
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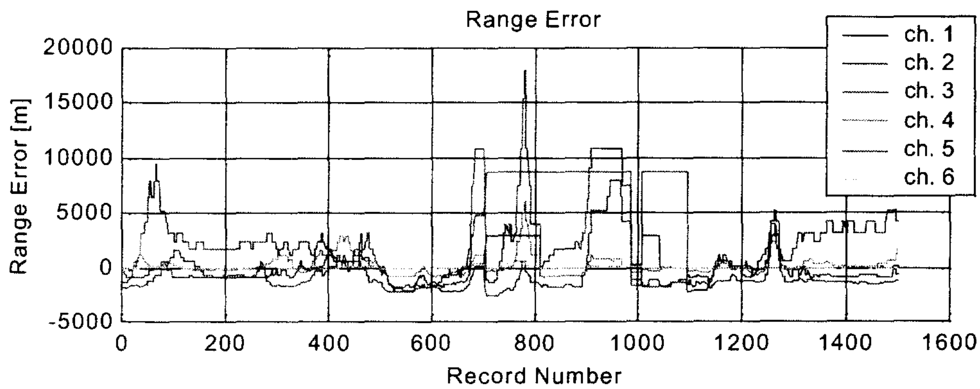
-120
0
200
400
600
800
1000
1200
1400
1600
Record Number
ch. 7
ch. 8


Range Error
Range Error [m]
20000
15000
10000
5000
0
-5000
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400
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800
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1400
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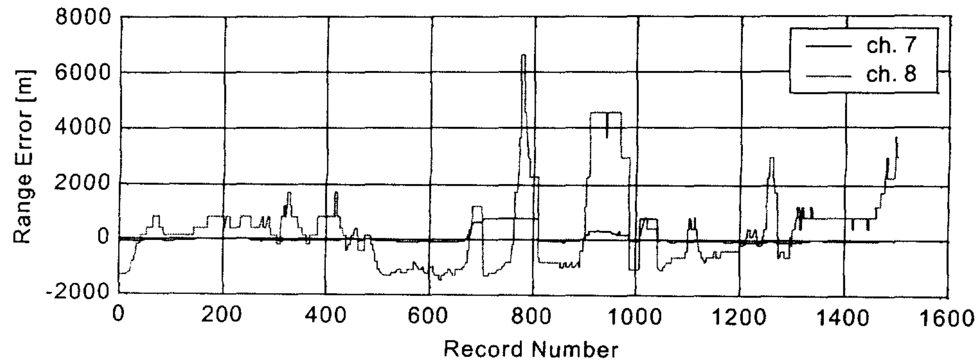
ch. 4
ch. 5
ch. 6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200
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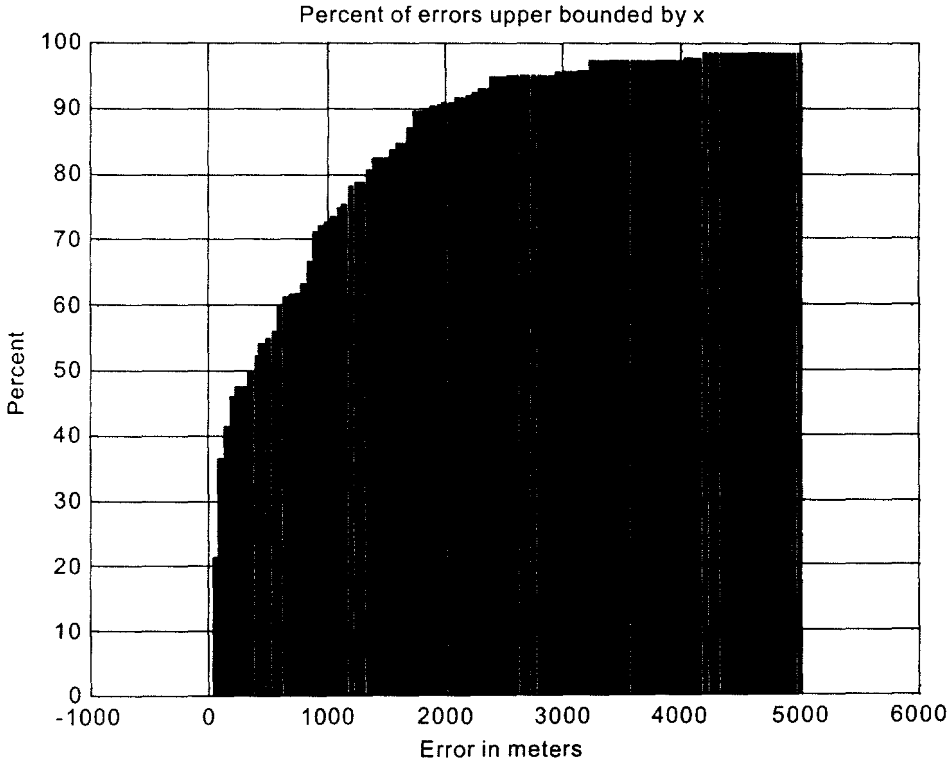
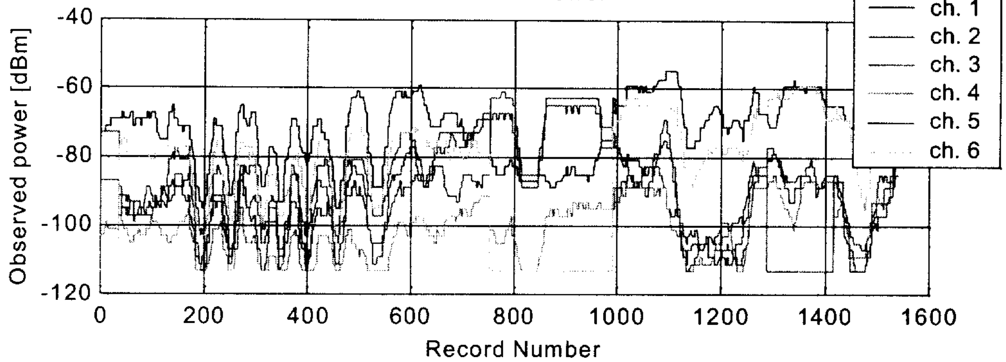
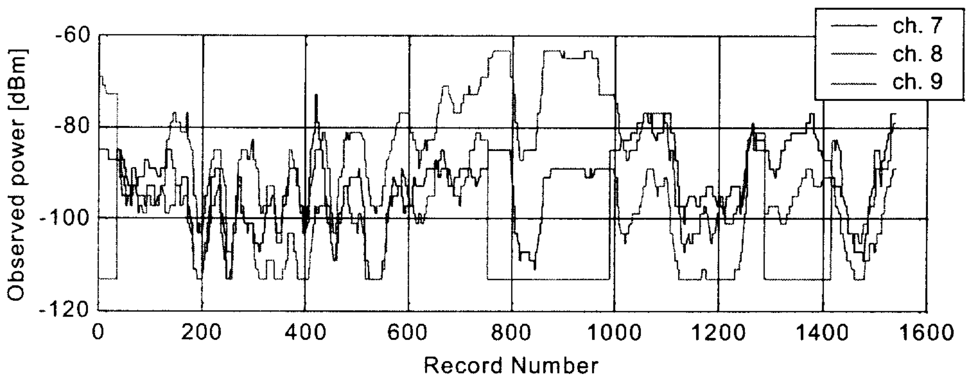
1000
1200
1400
1600
Record Number
ch. 7
ch. 8


Percent of errors upper bounded by x
Percent
Error in meters
100
90
80
70
60
50
40
30
20
10
0
-1000
0
1000
2000
3000
4000
5000
6000

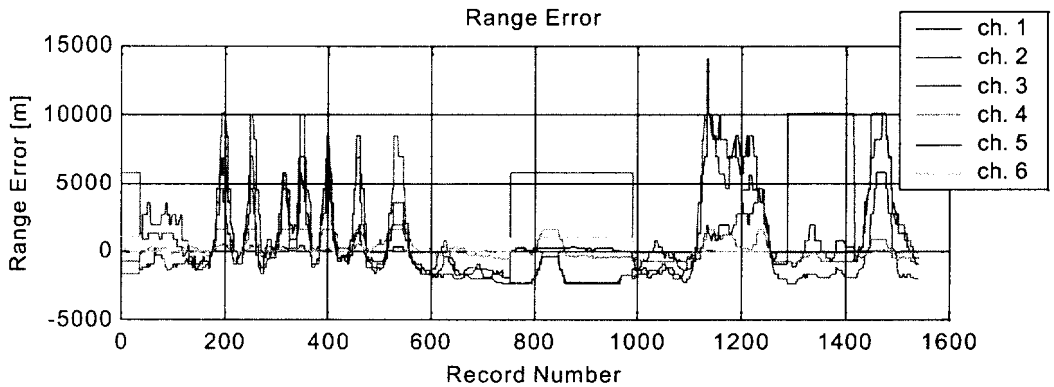
Test # 8

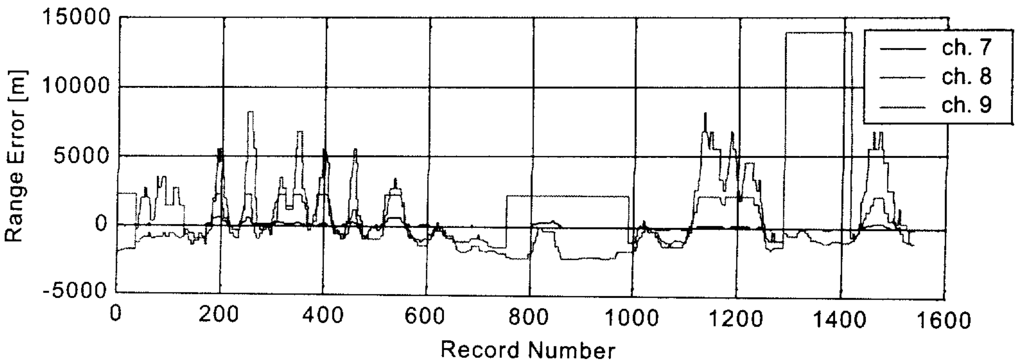
Observed Power

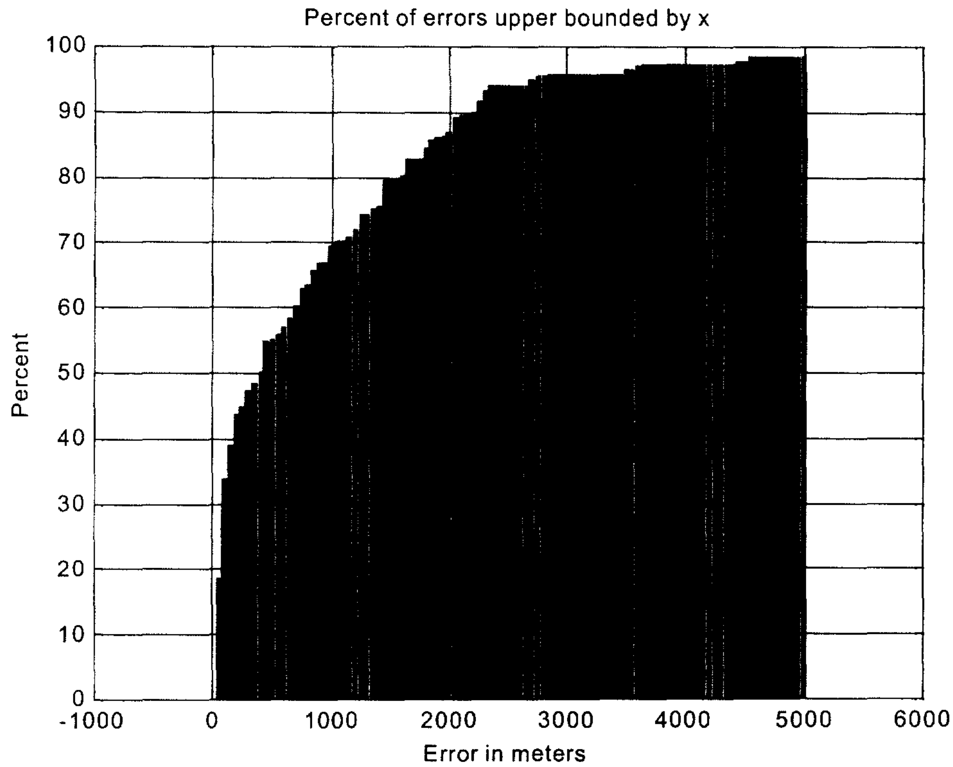




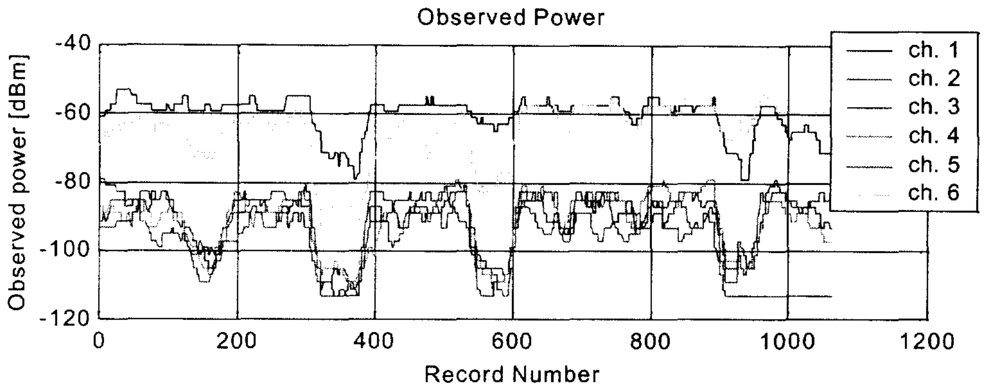
Range Error

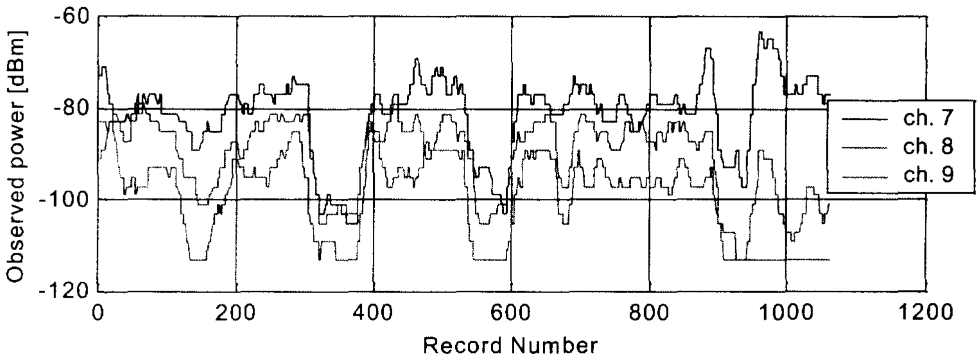


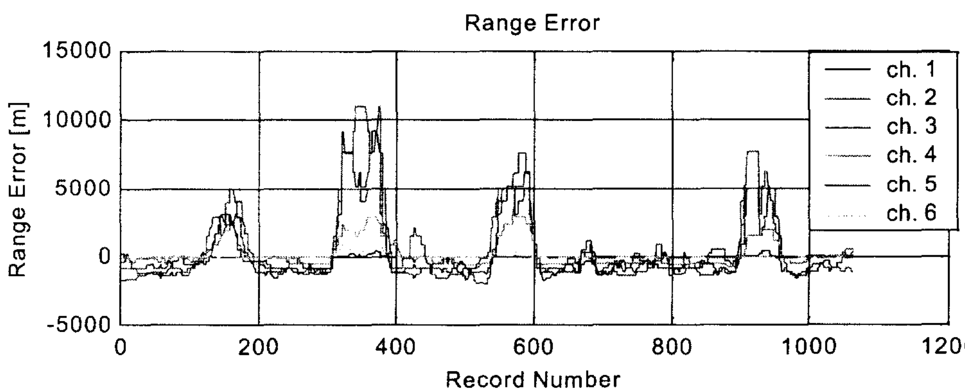


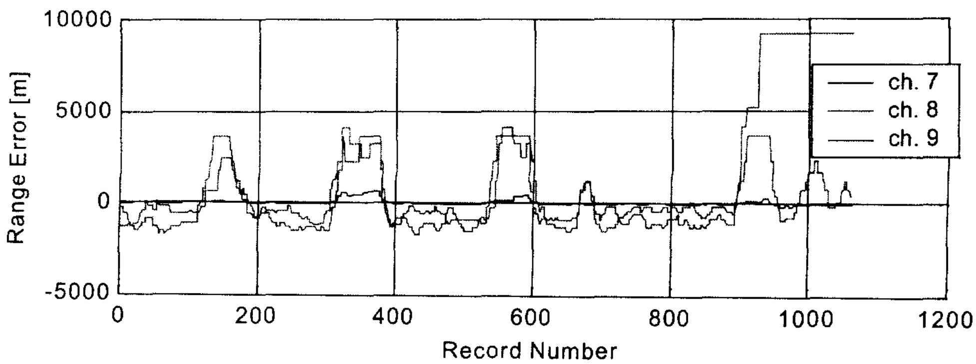

Percent of errors upper bounded by x
Percent
Error in meters
100
90
80
70
60
50
40
30
20
10
0
-1000
0
1000
2000
3000
4000
5000
6000

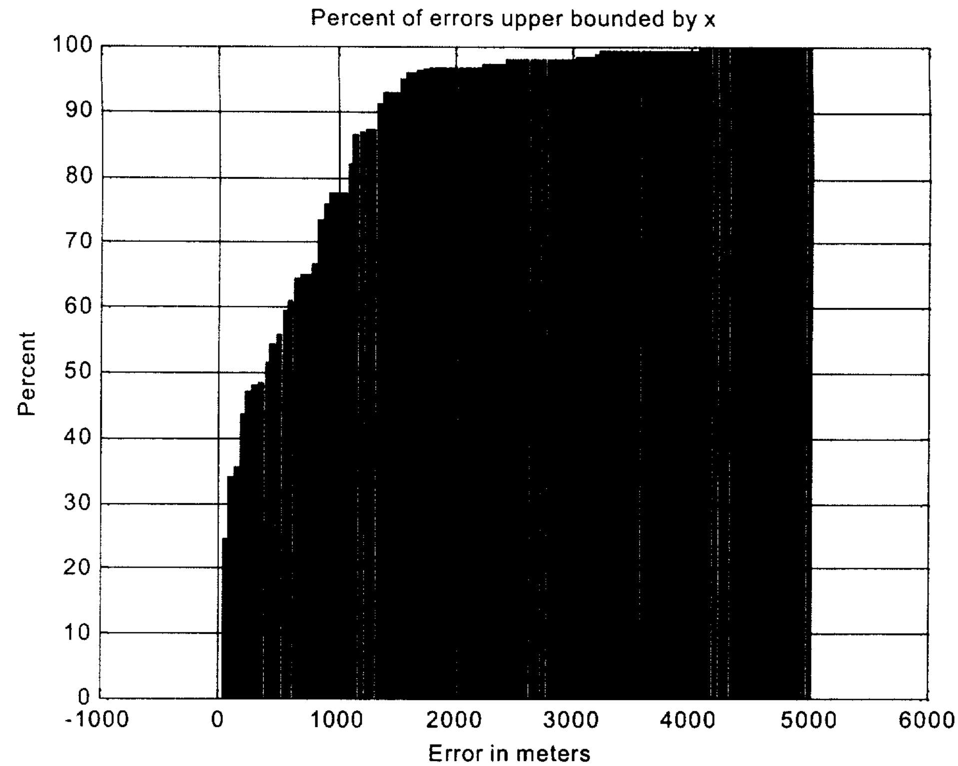
Test #9


Observed Power
Observed power [dBm]
-40
-60
-80
-100
-120
0
200
400
600
800
1000
1200
Record Number
ch. 1
ch. 2
ch. 3
ch. 4
ch. 5
ch. 6


Observed power [dBm]
-60
-80
-100
-120
0
200
400
600
800
1000
1200
Record Number
ch. 7
ch. 8
ch. 9

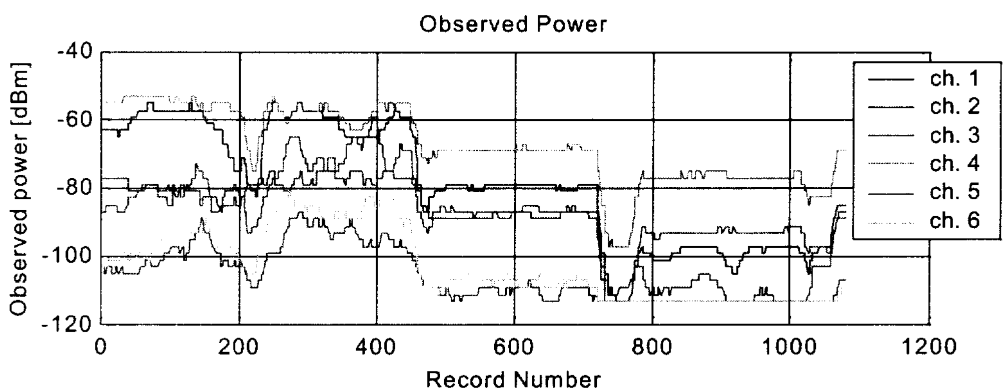

Range Error
Range Error [m]
15000
10000
5000
0
-5000
0
200
400
600
800
1000
1200
Record Number
ch. 1
ch. 2
ch. 3
ch. 4
ch. 5
ch. 6


Range Error [m]
10000
5000
0
-5000
0
200
400
600
800
1000
1200
Record Number
ch. 7
ch. 8
ch. 9

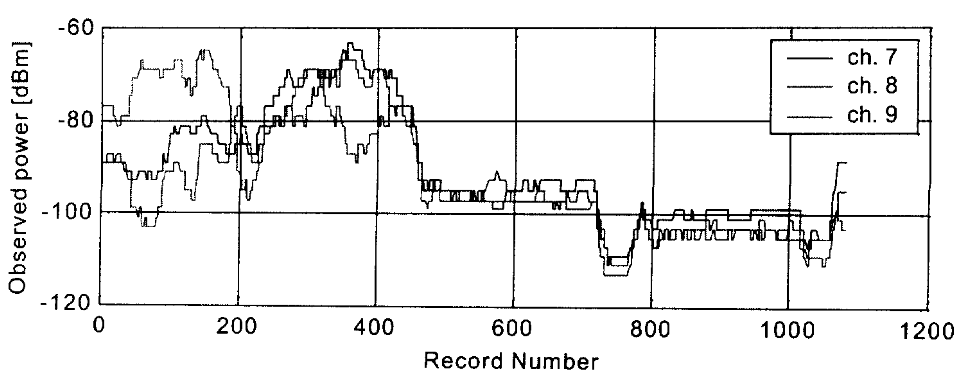


Percent of errors upper bounded by x
Percent
Error in meters

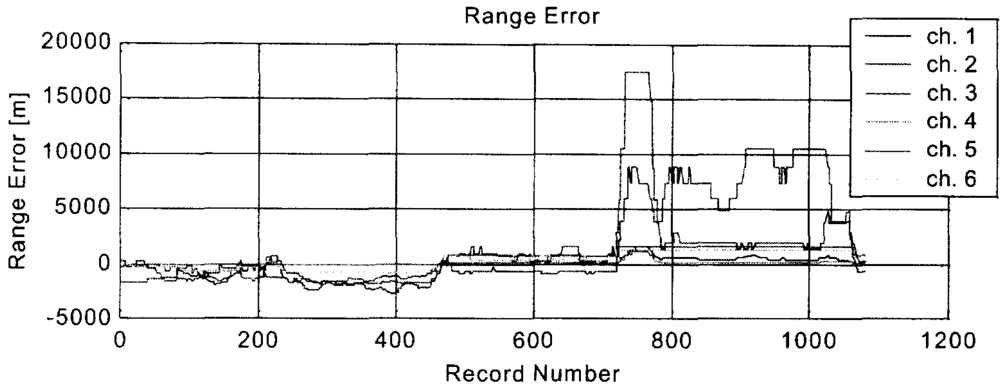
Test # 10

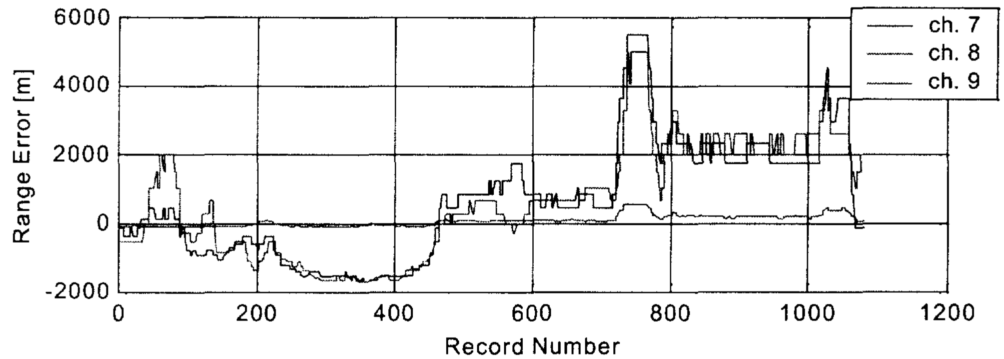


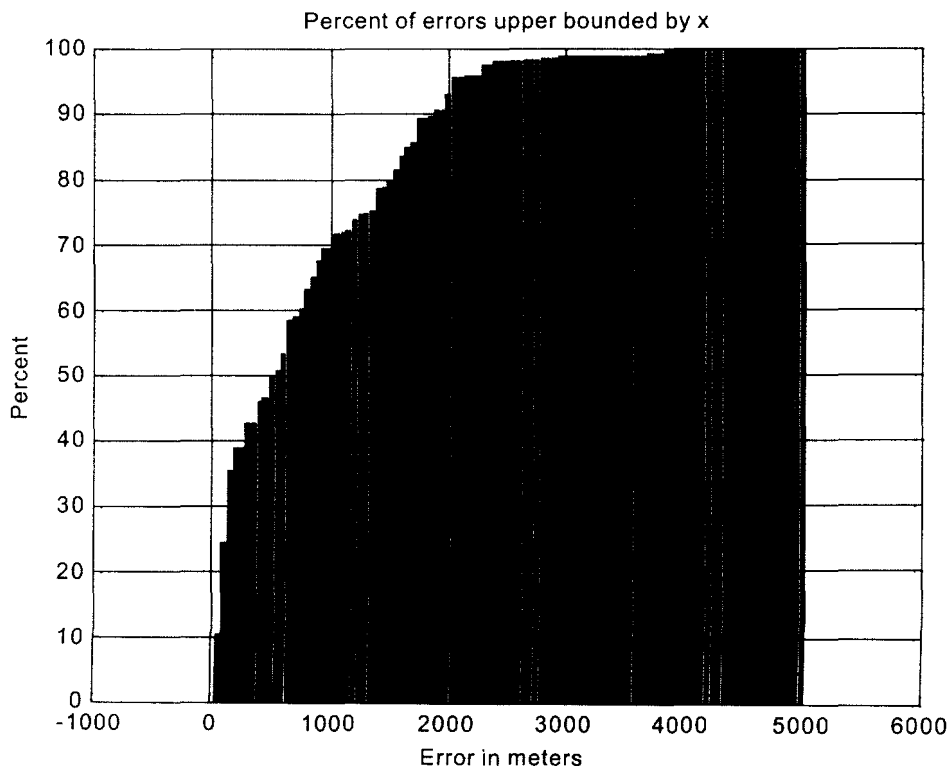
Observed Power
Observed power [dBm]
Record Number
ch. 1
ch. 2
ch. 3
ch. 4
ch. 5
ch. 6



Observed power [dBm]
Record Number
ch. 7
ch. 8
ch. 9

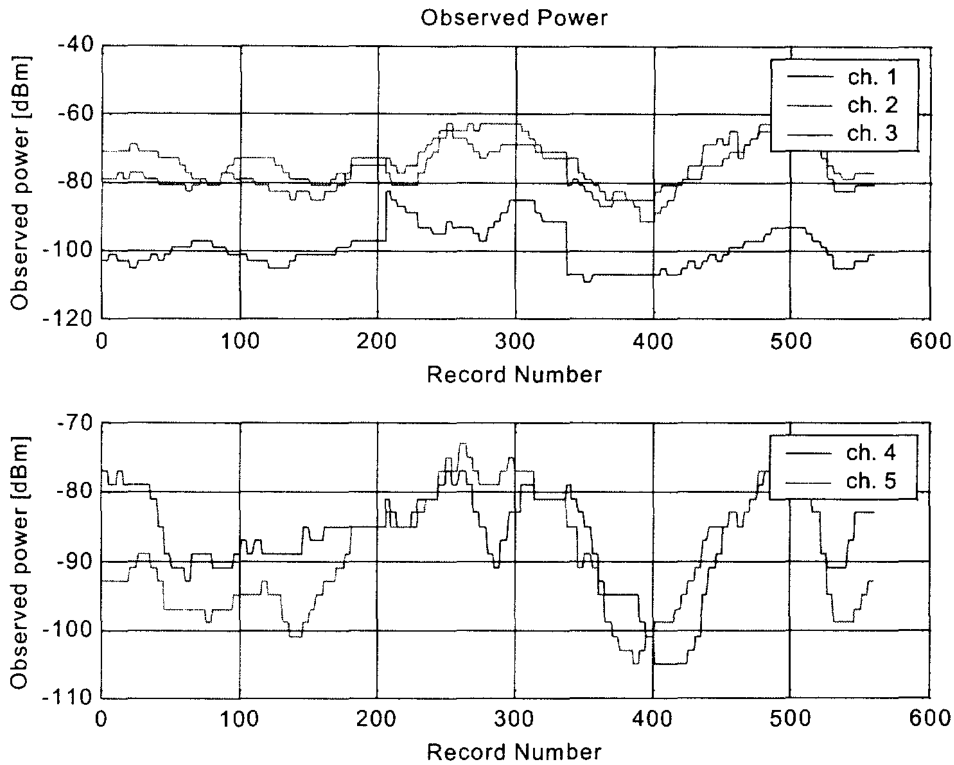

Range Error
Range Error [m]
Record Number
ch. 1
ch. 2
ch. 3
ch. 4
ch. 5
ch. 6

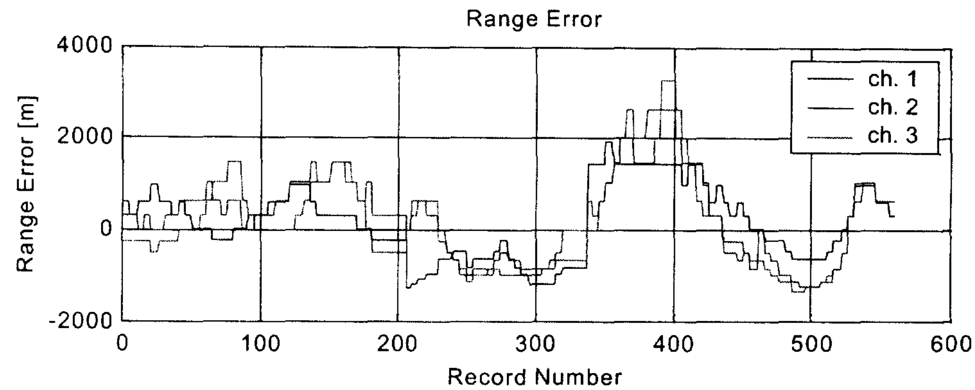

Range Error [m]
Record Number
ch. 7
ch. 8
ch. 9

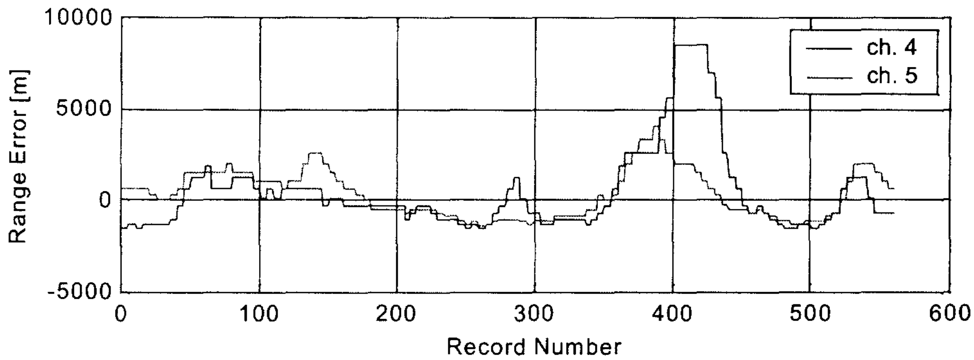

Percent of errors upper bounded by x
Percent
Error in meters

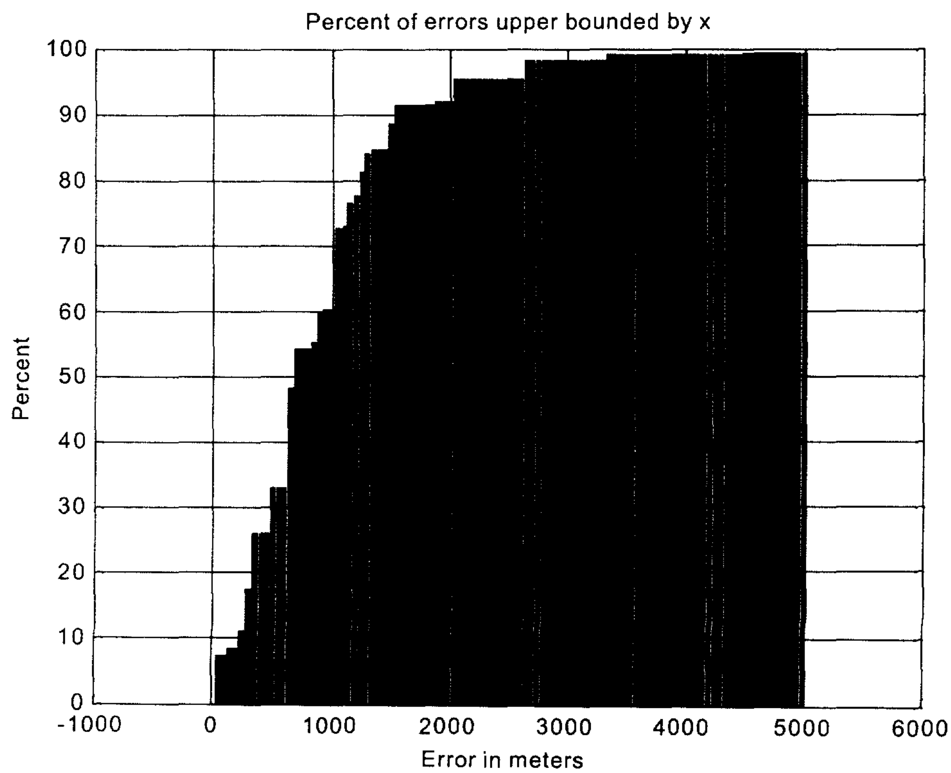
Test # 11

The 11th test was recorded in a car, which was driven 15m forward and backward in an open area within a distance of 50m from a three story buildings.



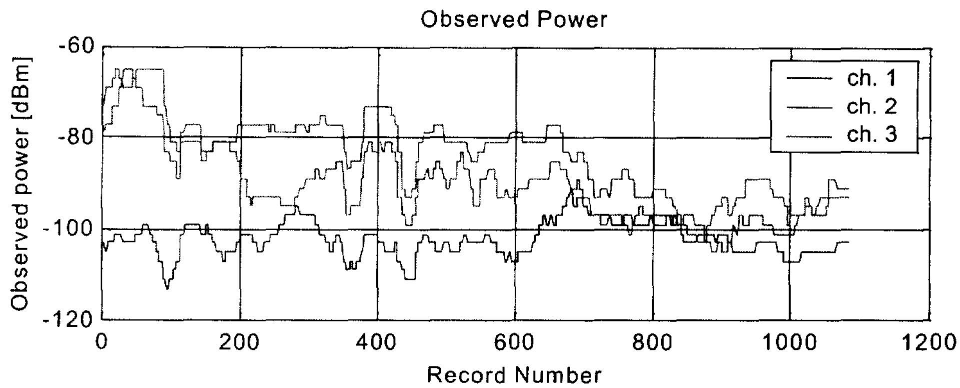

Range Error
Range Error [m]
Record Number
ch. 1
ch. 2
ch. 3
4000
2000
0
-2000
0
100
200
300
400
500
600

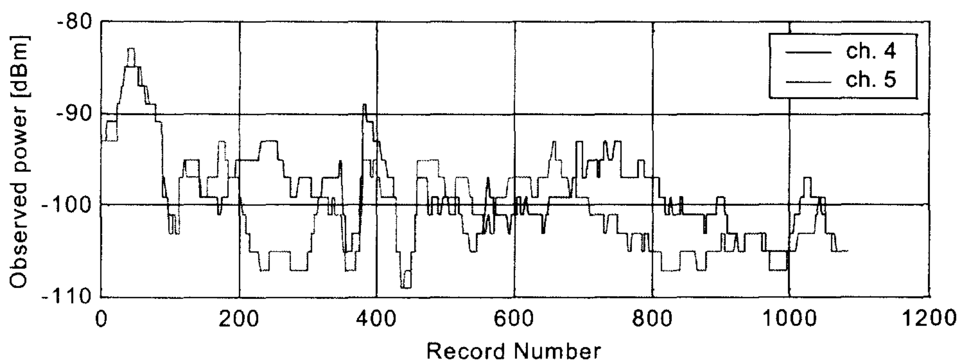

Range Error [m]
Record Number
ch. 4
ch. 5
10000
5000
0
-5000
0
100
200
300
400
500
600

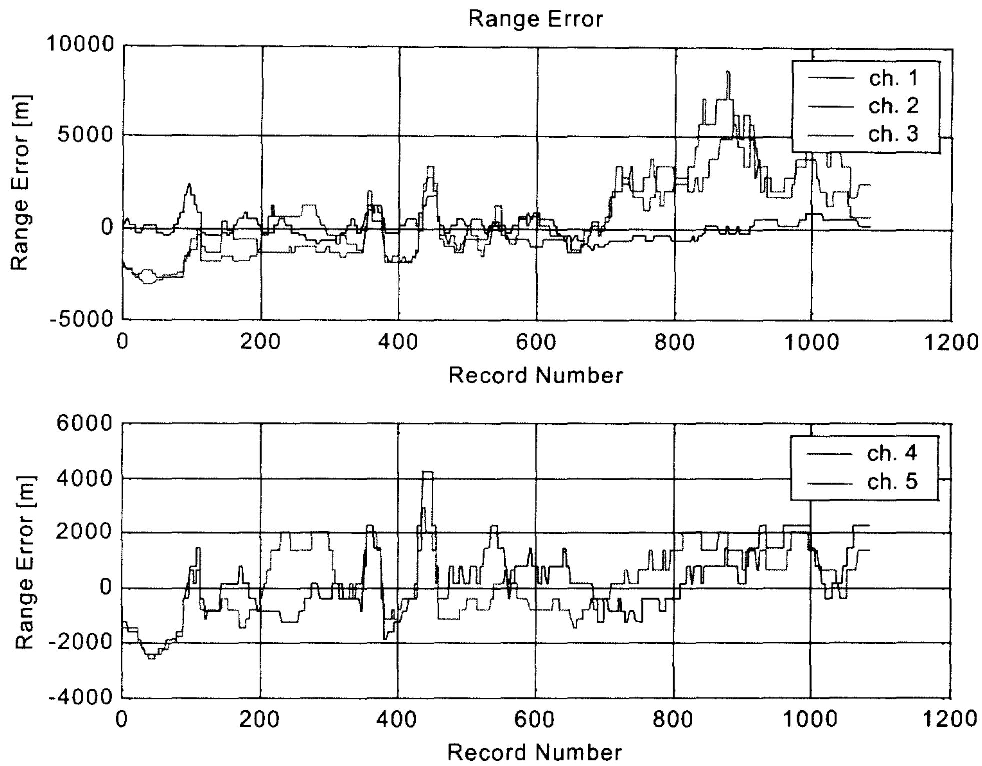

Percent of errors upper bounded by x
Percent
Error in meters
100
90
80
70
60
50
40
30
20
10
0
-1000
0
1000
2000
3000
4000
5000
6000

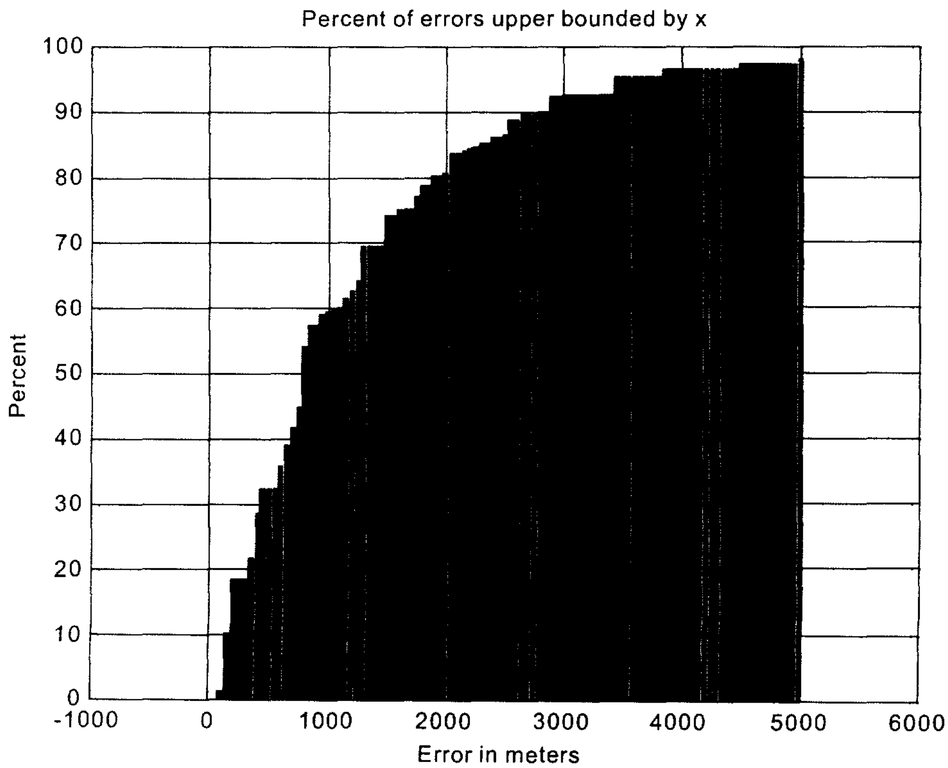
Test # 12

The 12th test was recorded while driving a car in a circle with a 60' radius, within the parking lot of a 5 story building.






Range Error
Range Error [m]
Record Number
ch. 1
ch. 2
ch. 3
ch. 4
ch. 5


Percent of errors upper bounded by x
Percent
Error in meters

3.8. Summary of results

	Test No. 1	Test No. 2	Test No. 3	Test No. 4	Test No. 5	Test No. 6
67% of range errors were below:	**760 meter**	**810 meter**	**1008 meter**	**1264 meter**	**680 meter**	**745 meter**
95% of range errors were below:	**2100 meter**	**3276 meter**	**2627 meter**	**2203 meter**	**1455 meter**	**1442 meter**

	Test No. 7	Test No. 8	Test No. 9	Test No. 10	Test No. 11	Test No. 12
67% of range errors were below:	**854 meter**	**951 meter**	**803 meter**	**888 meter**	**1027 meter**	**1277 meter**
95% of range errors were below:	**2538 meter**	**2696 meter**	**1458 meter**	**2040 meter**	**2046 meter**	**3442 meter**

4. TRANSLATION OF OBSERVED POWER FLUCTUATIONS TO DISTANCE ERRORS

The received signal power in a cellular environment is essentially related to the distance between the transmitter and receiver. A widely accepted model defining the statistical average of the propagation loss is a model proposed by Hata and Okumura. According to this model the received power is proportional to $R^{-\gamma}$, where R is the transmitter-receiver range and a typical value of γ is 3.3.

Consider two cases. In the first case the received power is P_1 and in the second case the received power is P_2. Using the Hata-Okumura model we get the following ratio between the corresponding distances,

$$K \equiv \frac{P_2}{P_1} = \left(\frac{R_1}{R_2}\right)^{\gamma}$$

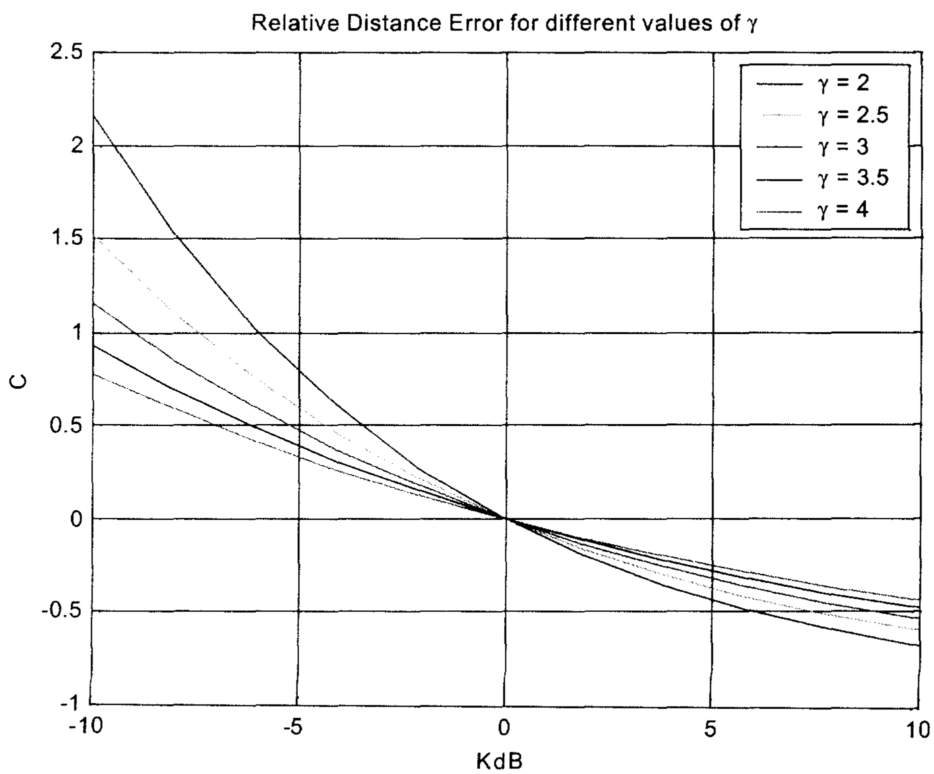
It follows that

$$\begin{aligned} K_{dB} &\equiv 10\log_{10} K = 10\gamma \log_{10}(R_1/R_2) \\ R_1/R_2 &= 10^{K_{dB}/10\gamma} \\ \Delta R &\equiv R_2 - R_1 = R_1(10^{-K_{dB}/10\gamma} - 1) \end{aligned}$$

Define

$$C \equiv (10^{-K_{dB}/10\gamma} - 1) = \frac{\Delta R}{R_1}$$

Some typical values of C are plotted below.



5. Position Determination Using Distance Measurements

5.1. Abstract

A Mobile Station position may be determined by using distance measurements from a set of base-stations whose position is known precisely.

This section derives the position determination error statistics as a function of the distance error statistics.

5.2. Definition of Variables

Assume that M base stations provide noisy distance measurements. Each measurement is provided with a quality factor that can be used as an estimate of the measurement variance. The distance measurement of the i-th base station is given by

$$\begin{aligned} m_i &= \sqrt{(x_t - x_i)^2 + (y_t - y_i)^2} + n_i = h_i(x_t, y_t) + n_i \qquad i = 1,2,\cdots M \\ h_i(x_t, y_t) &\equiv \sqrt{(x_t - x_i)^2 + (y_t - y_i)^2} \end{aligned} \tag{1}$$

Where $(x_t, y_t), (x_i, y_i)$ are the cellular phone coordinates and the base- station coordinates, respectively. The error in the i-th measurement is denoted by n_i.

According to the Least Squares principle the estimate of the phone coordinates is obtained by minimizing the cost function

$$Q = \sum_i \left| m_i - h_i(x_t, y_t) \right|^2 \frac{1}{\sigma_i^2} \tag{2}$$

Where σ_i^2 denotes the variance of the i-th measurement.

Since the cost function is a non-linear function of the phone coordinates it can be minimized using a few iterations.

Let's write the first order Taylor expansion of $h_i(x_t, y_t)$:

$$h_i(x_t, y_t) \cong h_i(x_k, y_k) + \begin{bmatrix} \frac{\partial h_i}{\partial x_t} & \frac{\partial h_i}{\partial y_t} \end{bmatrix} \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix},$$
$$\Delta x \equiv x_t - x_k \qquad (3)$$
$$\Delta y \equiv y_t - y_k$$

Note that

$$\frac{\partial h_i}{\partial x_t} = \frac{(x_t - x_i)}{\sqrt{(x_t - x_i)^2 + (y_t - y_i)^2}} = \cos\theta_i$$
$$\frac{\partial h_i}{\partial y_t} = \frac{y_t - y_i}{\sqrt{(x_t - x_i)^2 + (y_t - y_i)^2}} = \sin\theta_i \qquad (4)$$

Define the vectors and matrices

$$\Theta \equiv [m_1 \quad m_2 \quad \cdots \quad m_M]^T$$
$$\mathbf{h} \equiv [h_1 \quad h_2 \quad \cdots \quad h_M]^T$$
$$\mathbf{W} \equiv \mathrm{diag}\{\sigma_1^{-2} \quad \sigma_2^{-2} \quad \cdots \quad \sigma_M^{-2}\}$$
$$\Delta \mathbf{z} \equiv [\Delta x \quad \Delta y]^T \qquad (5)$$
$$\mathbf{J} \equiv \begin{bmatrix} \frac{\partial h_1}{\partial x_t} & \frac{\partial h_1}{\partial y_t} \\ \frac{\partial h_2}{\partial x_t} & \frac{\partial h_2}{\partial y_t} \\ \vdots & \vdots \\ \frac{\partial h_M}{\partial x_t} & \frac{\partial h_M}{\partial y_t} \end{bmatrix} = \begin{bmatrix} \cos\theta_1 & \sin\theta_1 \\ \cos\theta_2 & \sin\theta_2 \\ \vdots & \vdots \\ \cos\theta_M & \sin\theta_M \end{bmatrix}$$

Now equation (2) become

$$Q = [\Theta - \mathbf{h}(x_t, y_t)]^T \mathbf{W} [\Theta - \mathbf{h}(x_t, y_t)] \qquad (6)$$

Using a first order Taylor expansion we get

$$Q = [\Theta - \mathbf{h}(x_k, y_k) - \mathbf{J}\Delta\mathbf{z}]^T \mathbf{W} [\Theta - \mathbf{h}(x_k, y_k) - \mathbf{J}\Delta\mathbf{z}] \tag{7}$$

The minimization of the cost function with respect to $\Delta\mathbf{z}$ yields

$$\Delta\mathbf{z} = \left(\mathbf{J}^T \mathbf{W} \mathbf{J}\right)^{-1} \mathbf{J}^T \mathbf{W} (\Theta - \mathbf{h}(x_k, y_k)) \tag{8}$$

Thus, we can start from an initial guess of the phone location, and using (7) obtain a better estimate of the location. This can be repeated several times until the cost function is minimized. An update of **J** and **h** is required in each iteration.

This method is known as Gauss-Newton method. It is usually fast and effective. However, other methods are also available.

5.3. The FIX Covariance

Assume that the final fix estimate is close to the true location. In that case we have

$$\begin{aligned}
\Theta - \mathbf{h}(x_k, y_k) &= \mathbf{n} \\
\Delta\mathbf{z} &= \left(\mathbf{J}^T \mathbf{W} \mathbf{J}\right)^{-1} \mathbf{J}^T \mathbf{W} \mathbf{n} \\
\mathrm{E}\{\mathbf{n}\mathbf{n}^T\} &= \mathbf{W}^{-1} \\
\mathrm{E}\{\Delta\mathbf{z}\Delta\mathbf{z}^T\} &= \left(\mathbf{J}^T \mathbf{W} \mathbf{J}\right)^{-1} \mathbf{J}^T \mathbf{W} \mathrm{E}\{\mathbf{n}\mathbf{n}^T\} \mathbf{W}\mathbf{J} \left(\mathbf{J}^T \mathbf{W} \mathbf{J}\right)^{-1} = \left(\mathbf{J}^T \mathbf{W} \mathbf{J}\right)^{-1}
\end{aligned} \tag{9}$$

Where **n** is the measurement error vector. The computation of **J** is done using equation (5).

While this covariance was computed for the Gauss-Newton method it also reflects the error covariance for other algorithms such as a simple grid search.

A simple way to denote the fix accuracy is given by the longer semi-axis of the 67% uncertainty ellipse denoted here by a. Denote by λ_1 the largest eigen-value of the covariance matrix then

$$k = -2\ln(1 - P_e) = 2.2 \qquad (10)$$
$$a = \sqrt{k\lambda_1}$$

5.4. Numerical Examples

All propagation models (e.g. Hata, Hata-Okumura, Lee, COST-231) aim at estimating the median of the propagation path loss between the transmitter and the receiver. According to [2] page 106, Figure 3.17 actual RSSI measurements exhibit received signal strength standard deviation of 11.8 dB for cellular frequencies.

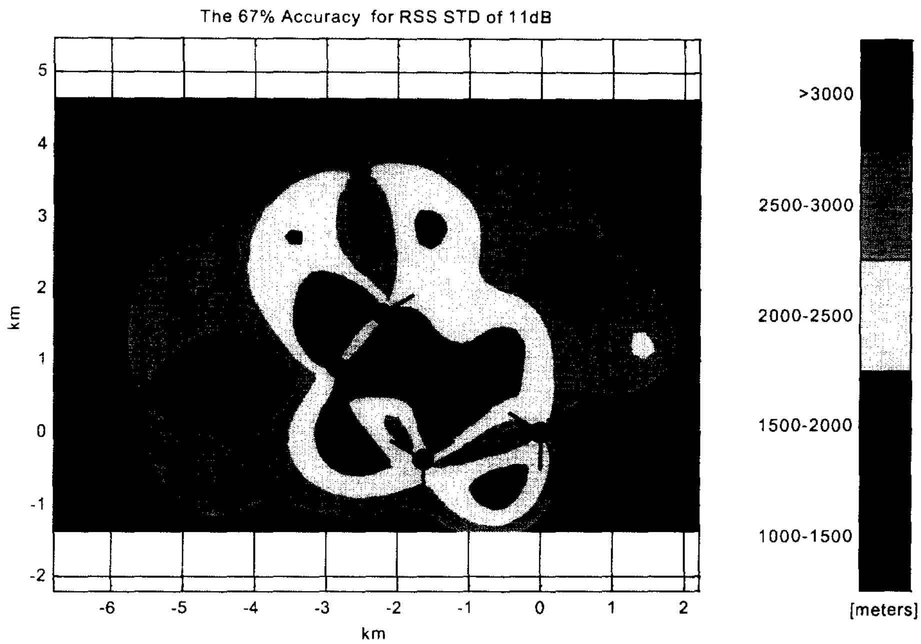
Using these results we ran an analysis of the best position estimate when 12 cell sites are received by a phone, which may occur in some outdoors environments but is unlikely in indoors environments. The best position estimate may be determined by table look-up (comparison of measured results vs. recorded results) or by iterative algorithmic computation.

In this scenario of the mobile station surrounded by 12 cell sites, distances between sites were less than 1 mile. We use the above standard deviation of 11.8 dB in the received signal strength. Using the mathematical derivation provided in Appendix A, it can be shown that typically the error will be worse than 1,550 meters for 67% of the calls. Practical implementation in realistic environments will lead to even higher errors.

Since these results already prove to be unusable, it is pointless to analyze the additional degradation in positioning accuracy resulting from the contribution of phone measurement accuracy, phone antenna directionality and variations in the path loss characteristics in different directions.

In conclusion, the simulations and analysis prove that RSSI is a highly undesirable technique to use for accurate location of a mobile handset.

As a further illustration, we used an actual TDMA cell site deployment in the suburban area of Rehovot, Israel. Furthermore using specific test data obtained from the field we obtained the actual standard deviation of RSSI as measured. We then get the following accuracy plot.



5.5. References:

[1] D. J. Torrieri, "Statistical Theory of Passive Location Systems," IEEE Transactions on Aerospace and Electronic Systems, Vol. AES-20, No. 2, March 1992.

[2] Theodore S. Rappaport, *Wireless Communications, Priciples and Practice*, Prentice Hall, 1996.